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A METHOD TO OPTIMIZE THE FLOOD RETENTION CAPACITY FOR A MULTI-PURPOSE RESERVOIR IN TERMS OF THE ACCEPTED RISK

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ABSTRACT

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In this paper, a new method is proposed to calculate the flood retention capacity for a multi-purpose reservoir in terms of the accepted risk (stochastic path method — SPM). No information is required about the reservoir operating rules and hence this method is particularly valuable for the planning stage. A computational algorithm for SPM is presented and the results obtained to applying this method are discussed.

INTRODUCTION

Reservoirs operated to provide firm supply of water (or power) are usually kept at their maximal possible levels in order to have enough stock of water to be used in the drought periods. On the other hand, reservoirs operated to mitigate downstream-of-the-dam flooding are usually kept at their minimal possible levels in order to have enough room to accommodate an incoming flood hydrograph. When the reservoirs are operated to meet both targets, it is necessary to develop some criteria to balance the two opposing goals. This is a question difficult to be formulated as an optimization problem because it is not easy to reduce all the costs and benefits associated to any decision into a common scale. For example, the complexities of assessing monetary values to the social impacts caused by flooding are well known.

An approach to the problem is estimating the flood retention capacity at each time interval t, $FC(t)$, in terms of the accepted risk of downstream flooding, α, and then using $FC(t)$ as a constraint to the operation rules aimed to maximize the firm yield. The effective capacity of the reservoir which can be used for head and stocking purposes is given by the difference between the reservoir useful capacity R and the flood retention capacity: $f(t) = R - FC(t)$. No need to emphasize how essential is $f(t)$ to assess the

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utility or the priority of the corresponding investment and hence the authors believe that $f(t)$ must be studied even at a preliminary stage of planning, when the reservoir operation rules are still not known.

The objective of this paper is the presentation of a method — the stochastic path method (SPM) — to estimate $f(t)$.

PREVIOUS METHODS

Several procedures have been developed with the purpose of estimating $f(t)$ for a level of risk equal to α, $f_\alpha(t)$, and they can be grouped into the following two major approaches.

(A) Estimation of the relationship between cumulative inflow during T time units and T, to determine the so-called "critical duration", T_c, for which the required spare storage is maximal (e.g., Beard, 1963).

(B) Simulation of the reservoir behaviour assuming a given $f(t)$ function and specific operation rules (e.g., Eichert, 1979).

In case A, the probability of insufficient spare capacity, α, is usually considered equal to the risk of having an inflow during T_c time units greater than that adopted to compute $f(t)$. In other words $P[Z(T_c) > FC(T_c) + T_cM] = \alpha$, where $Z(T_c)$ is the cumulative inflow during T_c time units and M is the maximal safe release during a unity time interval. However, the risk of downstream flooding is equal to $P[Z(1) > FC(t) + M$ or $Z(2) > FC(t) + 2M$ or $\ldots Z(T_c) > FC(t) + T_cM$ or $\ldots]$ which is obviously greater than or equal to α. Therefore the actual risk is underestimated by procedure A.

In case B, a large number of inflow time series are simulated, assuming full knowledge of the operation rules, in order to assess for each $f(t)$ which is the associated risk α. The basic shortcomings of this method are its indirect nature (α is determined in terms of $f(t)$, instead of determining $f(t)$ for α) and the need for making full assumptions about the operation of the reservoir, which is rather unacceptable for the planning stage.

The drawbacks of procedures A and B have stimulated the development of methods which allow the direct and precise calculation of the retention capacity. Similarly to approach B, these methods are based on the use of the complete inflow series registered or synthesized, but, on the contrary, $f(t)$ is not found through a trial and error process. For example, Kelman et al. (1981) have shown how to calculate the flood retention capacity of a reservoir already in operation for power production. In this case, the reservoir operation rules, disregarding flood mitigation, were well known and could be considered by the algorithm.

The method proposed by the authors, and presented in the next section, makes no imposition to the outflow rate $y(t)$ while the reservoir actual useful storage $V(t)$ is smaller than $f(t)$. It is assumed that $y(t)$ is immediately made equal to the maximal safe release M whenever $f(t)$ is reached by $V(t)$ (assumption of no delay of response). In short, the reservoir acts as follows:

(1) If $V(t) > R \rightarrow y(t) > M$ — if the actual useful storage is higher than the useful capacity, the dam-safety procedures override maximal release constraint. In this case the downstream-of-the-dam flood event cannot be avoided.

(2) If $R > V(t) > f(t) \rightarrow y(t) = M$ — the actual release is equal to the maximal release to control the flood and to lower the reservoir storage to the $f(t)$ target as soon as possible.

(3) If $f(t) > V(t) \rightarrow$ no rules for $y(t)$.

The maximal storage $f(t)$ should be evaluated in such a way that

$P[\exists t, t \in (0, h)$ such that $V(t) > R] =$

$= P[\exists t, t \in (0, h)$ such that $y(t) > M] =$

$= P$[downstream flooding in the interval $(0, h)] \leq \alpha$, where h is the last time interval of the flood season.

THE STOCHASTIC PATH METHOD (SPM)

The SPM needs a sample of inflow time series, $\{X_i(t)\}$ with $t = 1, 2, \ldots, h$; and $i = 1, 2, \ldots, N$, where N is the number of years. Each annual sequence is considered as a random realization of a stochastic process with a complex internal structure. This means that the concept of random event is reserved to each annual sequence (stochastic path) of important magnitudes such as inflows, reservoir levels, etc., instead of defining a statistical distribution for each of these variables associated to each time, t.

This method includes two major stages:

(A) Determining for the annual sequence of inflows i an associated path of *maximal permitted volumes of stored water*, $\{S_i(t)\}$ with $t = 1, 2, \ldots, h$.

(B) Comparative analysis of the N stochastic paths obtained for $\{S_i(t)\}$ with $i = 1, 2, \ldots, N$ and definition of $f_\alpha(t)$.

These two steps can be performed according to the procedures described in the following two sections.

Inductive derivation of $S_i(t)$

Let us suppose that: (A) inflow sequence i is taking place; and (B) there is no harm in ending the flood season with the reservoir full: $S_i(h) = R$.

At the beginning of the $(h-1)$th time interval the maximal storage volume is:

$$S_i(h-1) = \max\{\min[R, R - X_i(h-1) + M], 0\} \quad (1)$$

In words, if the inflow $X_i(h-1)$ is greater than the acceptable release M, then the difference must be absorbed by the reservoir in the $(h-1)$ time interval. If $S_i(h-1) = 0$ this means that the reservoir useful capacity is not enough to attenuate the last time interval inflow of the ith sequence and consequently the actual outflow is larger than $M: y(h-1) = X_i(h-1) - R$.

At the beginning of the $(h-2)$th time interval the maximal storage volume is:

$$S_i(h-2) = \max\{\min[R, S_i(h-1) - X_i(h-2) + M], 0\} \tag{2}$$

In words, if the inflow $X_i(h-2)$ is greater than the acceptable release M, then the difference must be absorbed by the reservoir in the $(h-2)$ time interval *and* the end storage should be at most equal to $S_i(h-1)$ in order to warrant enough attenuation storage for the inflow fluctuations beyond the $(h-2)$ time interval. Again if $S_i(h-2) = 0$, the actual reservoir yield $y(h-2) = X_i(h-2) - S_i(h-1)$ is larger than M.

In general:

$$S_i(t) = \max\{\min[R, S_i(t+1) - X_i(t) + M], 0\} \tag{3}$$

and let:

$$K_i = \begin{cases} 1 & \text{if } \exists\, t \in (0, \text{h}) \quad \text{such that } y(t) > M \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

If $K_i = 1$, then there is not enough volume to attenuate the ith inflow sequence.

Comparative analysis of the N stochastic paths

A set of N paths for $\{S_i(t)\}$ with $t = 1, 2, \ldots, h$ can be determined by applying the procedure of the previous section to a sample of N yrs. For the "no risk option", the function $f(t)$ is the "envelope" curve of the set of $\{S_i(t)\}$ paths to be "protected", i.e., for which no outflow exceedance will occur. Therefore, if the level of risk should be equal to α, one has to select a set of $J = \alpha N$ paths excluded from that "protection":

$$f_\alpha(t) = \min\{S_i(t)\} \text{ with } i \notin C(\alpha), \forall t$$

where $C(\alpha)$ is a set of those J paths. The number of $C(\alpha)$ sets that can be defined may be large, although smaller than $\dfrac{N!}{J!\,(N-J)!}$ because whenever $S_i(t) < S_k(t), \forall t$, the path k can only be included in $C(\alpha)$ if i is also included.

The optimal solution should minimize the inconvenience resulting from not using the full capacity of the reservoir which can be expressed by the maximization of:

$$G_\alpha = \sum_{t=1}^{h} G[f_\alpha(t)] \cdot l(t) \tag{5}$$

where $G[f_\alpha(t)]$ is a non-decreasing utility function of the maximal permitted storage and $l(t)$ is an appropriate function expressing the relative importance of the utility at different time units from $t = 1$ until $t = h$.

Furthermore, it sounds reasonable assuming that the optimal set $C(\alpha')$ for risk α', $C^*(\alpha')$, contains the optimal set for risk α, $C^*(\alpha)$, with $\alpha' > \alpha$, which has the additional advantage of simplifying the computational procedure to determine $C^*(\alpha)$ in terms of α.

A computational algorithm to determine $f_\alpha(t)$ in terms of α is presented in the next section using this assumption.

A computational algorithm for the stochastic path method

The objective of this algorithm is computing $f_\alpha(t)$ with $\alpha \leq \alpha_{max}$ using N inflow time series, parameters R and M, and functions $G(f)$, $l(t)$.

Step 1: Computing $S_i(t)$ with $t = 1, \ldots, h$ and $i = 1, \ldots, N$ using eqn. (3).

Step 2: Defining the set of paths which are relevant for this study, Ω, by disregarding any path j with $S_j(t) > q_\beta\,[S(t)]$ for all t and where $q_\beta[S(t)]$ is the β quantile of $S(t)$ with $\beta = \alpha_{max} + 1/N$.

Step 3: Determining the minimal risk, α_{min}, which is equal to:

$$\frac{\sum_{i=1}^{N} K_i}{N}$$

Step 4: The optimal set, $C^*(\alpha_{min})$ is given by the set of paths with $K_i = 1$. The envelope curve $f_{\alpha_{min}}(t)$ is given by:

$$f_{\alpha_{min}}(t) = \min\{S_i(t)\} \text{ with } i \notin C^*(\alpha_{min}) \text{ and } i \in \Omega$$

$$G_{\alpha\,min} = \sum_{t=1}^{h} G[f_\alpha(t)] \cdot l(t)$$

Let $\alpha = \alpha_{min}$

Step 5: Determining $C^*(\alpha')$ with $\alpha' = \alpha + 1/N$ by:

$$C^*(\alpha') = C^*(\alpha) \cup i^*$$

where i^* is the i-path not included in $C^*(\alpha)$, but included in Ω that will minimize:

$$G = \sum_{t=1}^{h} \min\{S_i(t) \text{ with } i \notin [C^*(\alpha) \cup i]\} \cdot l(t)$$

Hence:

$$f_{\alpha'}(t) = \min\{S_i(t)\} \text{ with } i \notin C^*(\alpha')$$

$$G^*_{\alpha'} = \sum^{H} G[f_{\alpha'}(t)] \cdot l(t)$$

Step 6: Set $\alpha = \alpha'$ and go back to step 5 until $\alpha' = \alpha_{max}$.

APPLICATIONS OF SPM

The algorithm presented in the previous section was applied to study the required flood retention capacity for a medium-size reservoir (Aguieira, with a catchment area of 3100 km^2 and capacity of 2×10^8 m^3) located at river Mondego (Portugal) and for a very large reservoir (Furnas, with a catchment area of 52,000 km^2 and capacity of 16×10^9 m^3) built at Rio Grande (Brazil).

Both reservoirs, Aguieira and Furnas, are associated to hydro-electric plants (with installed capacities of 270 and 1276 MW) and hence a higher flood retention capacity implies not just less effective capacity for the compensation of seasonal and interannual variation of inflows but also lower heads to generate power.

Furnas outflow should be lower than 4000 m^3 s^{-1} to avoid downstream flooding but its spillway maximal flow can reach 13,000 m^3 s^{-1} ("10,000 yrs flood"). The wet season lasts for 212 days ($h = 212$) from the beginning of October until the end of April and a historical record of 30 yrs ($N = 30$) of daily streamflow is available.

Aguieira outflow is added to the uncontrolled contribution of downstream tributaries (e.g., Rio Alva) and this sum should be lower than 1200 m^3 s^{-1} to avoid flooding the agricultural and urban areas of Coimbra (about 1500 ha and 200,000 inhabitants, respectively). The spillway maximal flow is equal to 2000 m^3 s^{-1}. The wet season runs from the middle of November until the middle of April and a historical record of 40 yrs ($N = 40$) of daily streamflows (upstream and downstream of Aguieira) is available.

The selection of α, probability of downstream-of-the-dam flooding, is a political decision implying a trade-off between rare but catastrophic losses and continuous expected economic gains generated through the operation of the reservoir (usually between 0.25 and 0.01).

Obviously, an important choice required by the application of this method concerns the number of hydrologic annual traces to be used which must be sufficiently high to obtain a reasonable number of paths included in the $C(\alpha)$, J, and, consequently, to guarantee a statistically significant estimate of $f_\alpha(t)$. Therefore, the lower is α, the higher should be such number.

The available historical records are not long enough and so a stochastic daily streamflow model (Kelman, 1980) is used to generate synthetic traces with different lengths for comparative purposes: 1000 yrs at Aguieira and 10,000 yrs at Furnas. Assuming $l(t) = 1$ and that $G[f(t)] = f(t)$, the computed results are presented in Fig. 1 showing that with $J < 100$ the non-smoothness of $f_\alpha(t)$ becomes too high. In the Furnas case, the envelope of all the 10,000 $[S_i(t)]$ is also shown.

CONCLUSIONS

SPM is a convenient method to optimize the required flood retention capacity as a time function for a specific level of risk. The main advantages

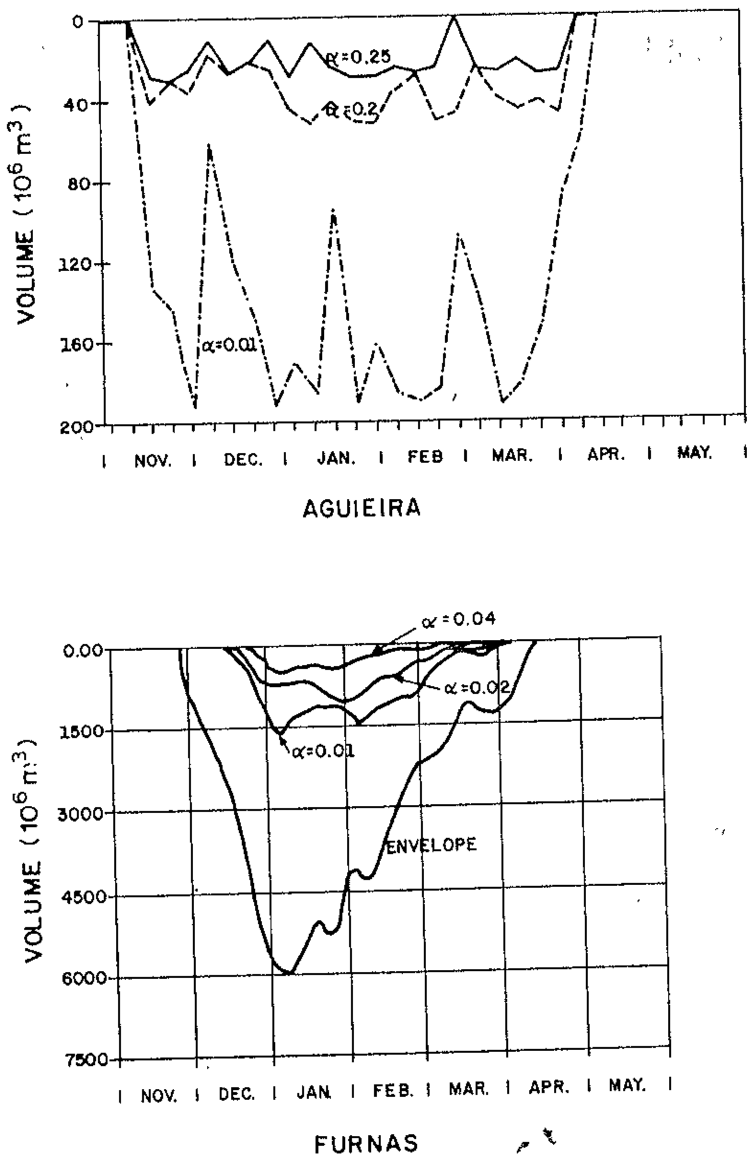

Fig. 1. The estimated $f_\alpha(t)$ for Aguieira and Furnas.

of this method are: (1) not using information about the reservoir operating rules; (2) making a correct statistical estimation of the flood risk; (3) considering the variation in time of the expected loss due to a capacity kept free for flood retention; and (4) allowing an easy computational implementation through the algorithm presented in this paper.

The major disadvantage of this method is the necessity of using a sufficiently long time series of streamflows often implying the adoption of a synthetic generator. (For the studied examples, $J = N \cdot \alpha$ should be higher than 100.) Obviously, the quality of the estimated results is largely dependent on the validity of such generator.

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LIST OF SYMBOLS

$C(\alpha)$	Set of paths $\{S_i(t),\ t = 1, 2, \ldots, h\}$ excluded from protection
$C^*(\alpha)$	Optimal set of paths for risk α
$f(t)$	Effective capacity of the reservoir $[R - FC(t)]$
$f_\alpha(t)$	Effective capacity of the reservoir associated with a probability of insufficient spare capacity α
$FC(t)$	Flood retention capacity at time interval t
$G(f)$	Non-decreasing utility function of the maximal permitted storage
G_α	Value of $G(f)$ for a risk α
h	Last time interval of the flood season
i	Inflow time-series index
J	Number of paths in $C(\alpha)$
K_i	Auxiliary variable
$l(t)$	Function expressing the relative importance of the utility at different time intervals
M	Maximum safe yield
N	Number of inflow time series (number of yrs of record)
$q_\beta[S(t)]$	β quantile of $S(t)$
R	Reservoir capacity
$S_i(t)$	Maximal permitted volume of stored water on time interval t, inflow time series i
t, τ	Time index
T_c	Critical duration or critical period
$V(t)$	Reservoir actual useful storage
$X_i(t)$	Inflow to the reservoir on time interval t, yr i
$Y(t)$	Outflow rate
$Z(t)$	Cumulative inflow during t time units

α	Probability of downstream-of-the-dam flooding
α_{max}	Maximum α
α_{min}	Minimum α
β	$\alpha_{max} + 1/N$

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