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A RISK-CONSTRAINED STOCHASTIC DYNAMIC PROGRAMMING APPROACH TO THE OPERATION PLANNING OF HYDROTHERMAL SYSTEMS

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Abstract - This work presents a new approach to the problem of finding operating strategies for a hydrothermal power generating system. The objective is to minimize the expected thermal generating costs subject to probabilistic constraints on the failure to supply the energy load. The problem is solved by a stochastic dynamic programing algorithm with nested reliability constraints from each stage to the end of the planning period. A decomposition approach is used to extend the methodology to the operation of two interconnected systems. Case studies with the South and Southeast Brazilian generating systems are presented and discussed.

INTRODUCTION

The Brazilian generating system is hydro-dominated, and characterized by large multi-year reservoirs [1]. Consequently, long-term operation planning studies should take into account the evolution of reservoir storages, the expected thermal generating costs and the risk of future energy shortages [2].

The problem to be solved is stochastic, since it is impossible to have perfect forecasts of the future inflow sequences. The existence of multiple interconnected reservoirs and the need for multi-period optimization characterize the problem as large-scale. Finally, it is also non-linear, due not only to non-linear thermal cost functions but also to the product outflow x head in the expression of hydroelectric production [3].

Therefore, it becomes necessary to develop methods able to approximate the solution of the operation planning problem. For the Brazilian system, this was accomplished by an aggregate representation of the hydroelectric system as one energy-storage reservoir and of the inflows as aggregate energy inflows [4]. This simplification allows the use of a stochastic dynamic programing (SDP) model to calculate the optimal operating strategy for each stage of the planning period [4].

The state variables of the SDP model are the stored energy in the aggregate reservoir at the beginning of each stage and the total energy inflow to the system during the previous stage. This last state variable represents the "hydrological trend" in the system and is necessary because the inflows in successive stages are highly correlated. The decision variable in each state is the amount of thermal generation. The SDP model uses a recursive algorithm to determine the optimal proportion of hydro and thermal generation in the system for each state and each stage of the planning period. The objective is to minimize the expected operation cost, composed of thermal costs plus penalties for failures in load supply. The recursive equation can be written as:

$$f_t(x_t,a_{t-1}) = \underset{k \epsilon K}{\mathrm{Min}}[CT(u_{tk}) + \underset{A_t|a_{t-1}}{E}\{\tfrac{1}{\beta}f_{t+1}(x_{t+1}(x_t, A_t,u_{tk}),A_t) + CD(d_t(x_t,A_t,u_{tk}))\}] \tag{1}$$

where:

- x_t is the energy storage at the beginning of stage t,
- a_{t-1} is the aggregate energy inflow during state t-1,
- k indexes the set of thermal generating decisions,
- K is the set of thermal generating decisions,
- u_{tk} is the k-th thermal decision at stage t,
- $CT(.)$ is the thermal cost function,
- $\underset{A_t|a_{t-1}}{E(.)}$ is the expected value over the inflows during stage t, A_t, conditioned by the inflow during stage t-1,
- $1/\beta$ is the discount factor,
- $x_{t+1}(.)$ is the system transition function,.
- $CD(.)$ is the deficit cost function,
- $d_t(.)$ is the deficit function.

The penalty function CD(.) associated to the energy shortages $d_t(.)$ in Expression (1) should ideally reflect the reduction in economical activities caused by the failure in load supply. However, such macro-economical effects are extremely difficult to quantify [5]. As a consequence, the adequacy of an operating strategy is in practice measured by the energy supply reliability it can provide [6]. In other words, operating planning studies aim to find operating strategies that minimize the expected thermal generating cost and satisfy given reliability constraints.

One possible approach to the solution of this problem is to use the deficit cost function as a parameter to ensure that the resulting operating strategies satisfy the target reliability constraints [7]. This penalty-augmented approach can be easily implemented through the following procedure:

a) Start with a given deficit cost function.
b) Calculate the operating strategy using the SDP recursion.

c) Simulate the system operation with a large sample of inflow sequences and estimate the supply reliability.

d) If the reliability level is equal to a pre-established value, within a certain tolerance, stop. Otherwise, modify the deficit cost function to higher or lower values depending of the reliability level calculated. Go to (b).

This procedure has been used for some years in the operation planning studies of the Brazilian system [8]. The reliability level corresponds to the risk of any energy shortage during the planning period. The adopted deficit cost function is linear, that is, there is only one parameter to be adjusted.

The objective of this work is to investigate the effect of this indirect representation of the reliability constraint on the optimality of the operating strategy and whether it can be replaced by an explicit risk-constrained SDP approach. In the next section, some methodological aspects of reliability-constrained reservoir optimization are briefly reviewed.

RELIABILITY-CONSTRAINED RESERVOIR OPTIMIZATION

As mentioned in the previous section, the objective of the operation planning problem could be stated as follows:

"Choose among all possible operating strategies the one that minimizes the expected thermal generating cost and satisfies given reliability constraints".

Given this statement, the optimality of the penalty-augmented procedure (a)-(d) discussed in the previous section can be questioned. For example, one could ask whether a non-linear deficit cost function could lead to a strategy with lower expected thermal cost which would still satisfy the reliability constraints.

This problem has been investigated in a slightly different context in Reference [9]. The problem in that case was to maximize the expected net benefits resulting from the operation of a single reservoir subject to a reliability constraint on the expected number of years in which the system fails to meet a target release. Through Lagrangian duality theory, it is shown in reference [9] that:

- the reliability constraint can be incorporated in the objective function in a similar way as the penalty-augmented approach discussed in the previous section. It is also shown that the adjustment of only one parameter in the penalty function is enough to ensure optimality.

- is terms of the reliability constraint, it does not matter in which stage of the planning period the failure occurs. This implies that the penalty function should not be affected by the discount factor $1/\beta$ used to evaluate the present value of the expected net benefit.

The first result suggests that penalty-augmented approaches are valid as solution procedures. The second result indicates that the specific procedure (a)-(d) presented in the previous section does not lead to an optimal solution, since deficit costs are discounted in that formulation.

However, an unexpected result appears if the penalty-augmented procedure is modified in accordance with the above conclusions. Since it does not matter in which stage of the planning period the failure occurs, "the optimal strategy tends to distribute the majority of the allowed failures to the early years, when benefits are worth more, by following a more risky police than that used in later years, when benefits are worth less" [9].

This result is certainly not acceptable in terms of the operation planning studies. It should be noted, however, that the solution is optimal in terms of the objectives stated at the beginning of this section. This implies that the modelling of the operating planning problem is not adequate.

The class of reliability constraints studies in Reference [9] was limited to the expected number of failures along the planning period. The results were later on extended to take into account the risk of any failure during the planning period [10], which corresponds to the reliability measure presently used in operation planning studies. It is shown in Reference [10] that constraints of this type can also be handled by penalty-augmented approaches, provided than an additional state variable, representing the number of failures up to the present stage is included in the model.

The operation strategies thus obtained indicate further inconsistencies in the modelling of the operation planning problem. For instance, it is shown in Reference [10], that, if the reliability constraint has been violated in the early stages of the planning period, the operating strategy in the following stages becomes extremely risky.

It is observed in Reference [10] that "it is rather unlikely that decision makers responsible for reservoir operation would be willing to implement strategies that sooner or later ignore the reliability constraint". It is also suggested that the proposed procedure could be used to test heuristic strategies.

This result is not acceptable in terms of the operation planning studies. Once again, the problem is to use an adequate modelling of the real life situation.

In the operation planning problem, the decision-making process is repeated for each state in each stage, looking towards the future operating costs, and forming a set of nested problems. To be coherent, the inclusion of reliability aspects in this process must be done in the same manner. In other words, what must be taken into account in each stage and state is the shortage risk from that point to the future. Failures that occurred in the past should not affect decisions for the future.

Therefore, in each stage there will be a reliability constraint that limits the shortage risks within a fixed period in the future to pre-established target risks, as shown in Figure 1. Although this formulation may be realistic, it has some disadvantages, which will be discussed later.

Another possible way of representing the reliability constraint is to establish targets on the shortage risk from every stage to the end of the planning period, as illustrated in Figure 2. This approach makes the problem easier to handle and is not a very strong simplification, since the operating strategies are revised each year.

The problem of minimizing the expected thermal generating costs subject to risk constraints of this type can be formulated as a sequential decisions problem and solved by a dynamic programing algorithm, as it will be seen in the next section.

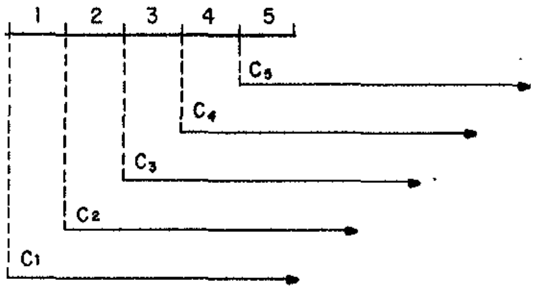

Figure 1 : Formulation of the nested problems with a fixed period in the future.

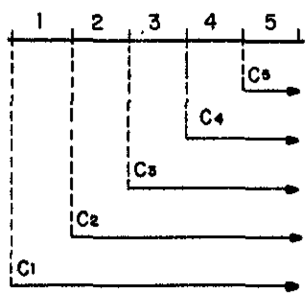

Figure 2 : Formulation of the nested problems with constraints limited at the end of the planning period.

THE RISK-CONSTRAINED SDP ALGORITHM

Evaluation of the shortage risk

The energy shortage probability from a stage t to the final stage N, associated to a state x_t can be defined as the union of two events : shortages in stage t and shortages in the future, as shown in Expression (2).

$$P(D_{t,N+1} > 0 | x_t) = P(D_{t,t+1} > 0 \cup D_{t+1,N+1} > 0 | x_t) \quad (2)$$

These two events are not mutually exclusive and by applying probability theory, Expression (2) can be written as follows:

$$P(D_{t,N+1} > 0 | x_t) = P(D_{t,t+1} > 0 | x_t) + P(D_{t+1,N+1} > 0 | D_{t,t+1} = 0; x_t) \cdot P(D_{t,t+1} = 0 | x_t) \quad (3)$$

A simple balance equation gives the stored energy (positive or negative) at the end of stage t for fixed initial state x_t, thermal generation u_{tk} and energy inflow A_t. Since the only random variable in this balance equation is the energy inflow, it can be stated that the probability distribution of the final stored energy at stage t is a function of the probability distribution of A_t.

In practice, the range of energy inflow at stage t is discretized into intervals, with middle points $a_t(i)$ and associated probabilities $p_t(i)$. This set can be divided in two subsets : AS_t, inflows that take the system to positive stored energy at the end of the stage, and AD_t, inflows that will take the system to a deficit situation, for a fixed thermal generation.

Considering this discretization scheme and representing the shortage probability from stage t to the end of the period, for a given state x_t, as the shortage risk from x_t, $R_t(x_t)$, Expression (3) can be rearranged as:

$$R_t(x_t) = \sum_{m \in AD_t} p_t(m) + \sum_{i \in AS_t} R_{t+1}(x_{t+1}(i)) \cdot p_t(i) \quad (4)$$

The recursive characteristic of the shortage risk shown in Expression (4) allows its evaluation for each state and stage in a SDP algorithm.

It should be remarked that, when using a reliability constraint involving a fixed period in the future, some computational problems will appear at this point. The shortage risk for this fixed period must be calculated for each state x_t in each stage t. However, it would also be necessary to calculate the shortage risk for smaller periods in order to be able to solve the problem at stages t-1, t-2 and so on.

The adoption of a two-dimension state variable in the model does not modify what has been exposed until now. The probability associated to an inflow $a_t(i)$ in stage t, given $a_{t-1}(j)$ in the previous stage will be represented by $p_t(j,i)$. The shortage risk will be represented by $R_t(x_t, a_{t-1})$.

Formulation of the Problem

By defining a set of nested reliability constraints composed of target values for the shortage risks from each stage to the end of the planning period, TR_t, the problem of minimizing the expected thermal generating costs subject to this risk constraint can be formulated as follows:

$$f_t(x_t, a_{t-1}) = \min_{k \in K} [CT(u_{tk}) + \underset{A_t | a_{t-1}}{E} \{ \tfrac{1}{\beta} f_{t+1}(x_{t+1}(x_t, A_t, u_{tk}), A_t) \}]$$

s.t. (5)

$$R_t(x_t, a_{t-1}(j)) = \sum_{m \in AD_t} p_t(j,m) + \sum_{i \in AS_t} R_{t+1}[x_{t+1}(x_t, a_t(i), u_{tk}), a_t(i)] \cdot p_t(j,i) \le TR_t$$

In each stage of the planning period, for each state of the system, the constrained optimization algorithm goes through the following steps:

a) Start with the lowest thermal decision as the optimal and with the expected cost of the previous decision as infinity.

b) Compute the present expected thermal generating cost.

c) Compute the expected shortage risk.

d) If the shortage risk is greater than the target risk and this is the maximum thermal generation, this is the optimal decision. Stop. Otherwise, go to (e).

e) If the shortage risk is greater than the target risk, increase the thermal generation and go to (b). Otherwise, go to (f).

f) If the expected cost for this decision is less than the cost of the previous one, increase the thermal generation and go to (b). Otherwise, go to (g).

g) If the shortage risk of the previous decision is less or equal to the target risk, then the previous decision is the optimal one. Stop. Otherwise, the optimal decision is the present one. Stop.

According to the target risks chosen, two different cases may occur, as shown in Figure (3).

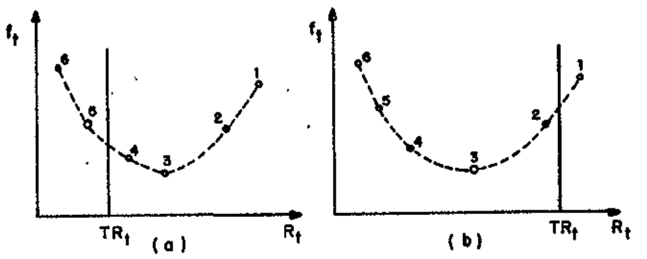

Figure 3 : Activity and non-activity of the risk constraint.

In case (a) the risk constraint is active. The optimal decision is # 5, although decision # 3 has a lower expected operating cost. In case (b), the constrained optimization algorithm will indicate decision # 3 as the optimal, since the risk constraint is not active.

Thus, it is clear that the use of adequate target risks for the system will have a strong influence on the resulting strategy.

Definition of the target risks

The natural risk of each stage represents the probability of energy shortages from that point to the end of the planning period, given an initial state of the system at the first stage, and considering no thermal generation in the period.

The natural risk values, $NR_t(x_1,a_0)$, are estimated by simulating the system operation, from the given initial state, considering only the hydro system and using a large sample of synthetic energy inflow sequences, according to Expression (6).

$$NR_t(x_1,a_0) = \sum_{i=1}^{L} \delta_{it}\cdot p_t(i) \text{ with } \delta_{it} \begin{cases} = 1 \text{ if } D_{t,N+1} > 0 \\ = 0 \text{ if } D_{t,N+1} = 0 \end{cases} \qquad (6)$$

where

- L is the number of sequences used in the simulation.
- δ_{it} is a 0-1 variable indicating energy shortages between stage t and the end of the period.
- $p_t(i)$ is the probability associated to an inflow $a_t(i)$ in stage t.

The risk constraint is obtained by a reduction of the natural risks. The natural risk curve is multiplied for the factor that reduces the natural risk of the first stage to an objective risk for the planning period, considered in the operation planning study.

It should be remarked that this is a heuristic procedure and that many others ways of defining the target risks may be established.

Application

The risk-constrained operating strategy was determined for the Southeast Brazilian generating system, in the 1982-1986 period, according to the previously described procedure.

The natural risk and target risk curves presented in Figure 4 were obtained considering state (51,7) in a (101,10) discretization grid as the initial state of the system in January/82. It was assumed an objective risk of 3% for the five years planning period.

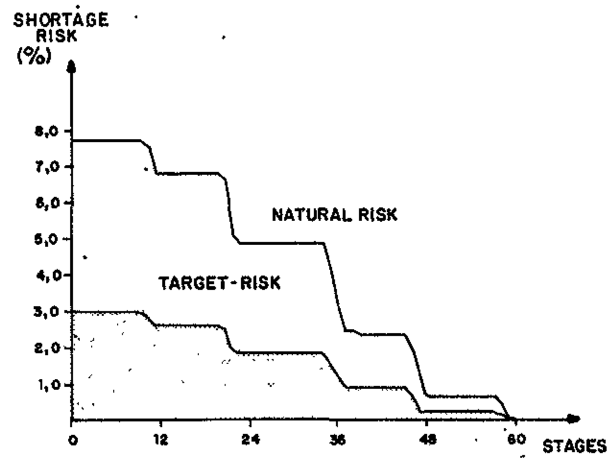

Figure 4 : Natural risk and target risk curves for the Southeast Brazilian system.

The operation strategy was obtained considering the reliability constraint formed by the target risk values. Figure 5 shows the shortage risk surface calculated for the first stage of the planning period.

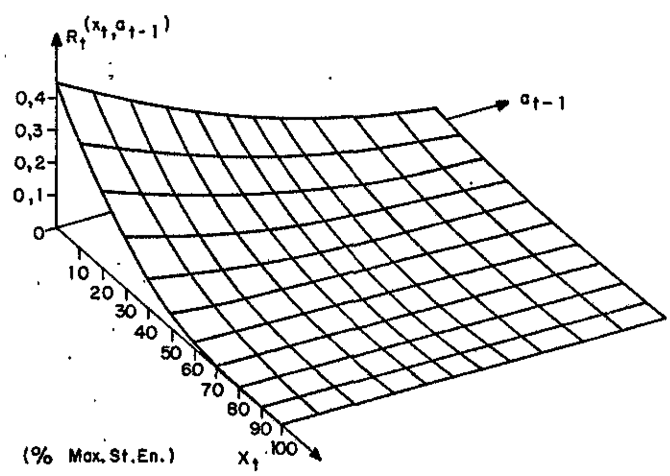

Figure 5 : Shortage risk surface of the first stage.

By comparing the shortage risk values of the resulting strategy with the target risks, one can notice that there are two distinct regions : an unsafe operation region, in which the reliability constraint is violated; and a safe operation region, in which the shortage risks are below the target risk. Figure 6 illustrates this situation in the first stage.

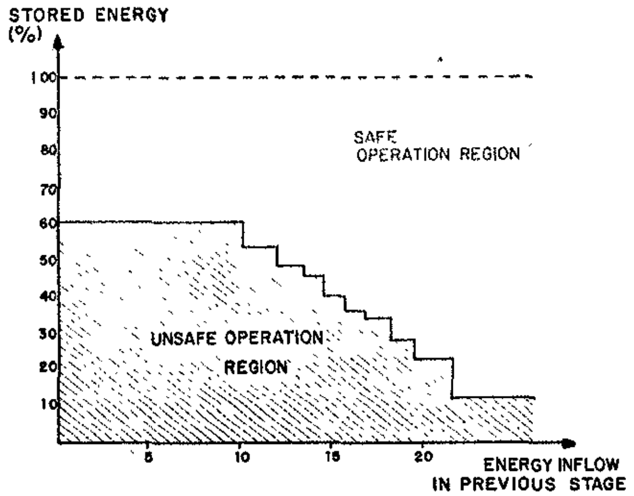

Figure 6 : Unsafe and safe operation regions for the first stage.

The usefulness of these regions will be seen in the next section.

EXTENSION FOR TWO INTERCONNECTED SYSTEMS

The decomposition approach

Dynamic programing optimization models have been widely used in energy management studies for power generating systems. The representation of the stochastic characteristics of some variables in models of this class imposes severe constraints in the modelling technique, due to a considerable increase in memory requirements and processing times, in nowadays conditions. However, experience has demonstrated that significant accuracy can be obtained in operating strategies of hydro-dominated hydrothermal generating systems, when these stochastic characteristics are represented in the model.

Therefore, most real systems are represented by aggregate models in order to make SDP algorithms applicable, specially when bi-dimensional state variables are used. Even though, there may be situations in which the characteristics of the real system, as a whole, do not perfectly fit the basic assumptions of the simplified models. For example, the aggregate model presently used in Brazil suposes a perfect integration of the transmission network and assumes no hydrological diversity between the river basins that compose the real system.

A decomposition approach has been succesfully used to determine operating strategies of systems in which aggregate models can not be directly applied. In this approach, the optimization of the whole system is reached from isolated optimizations of the subsystems in which it can be decomposed.

This approach comprises the use of an iterative procedure in which separate strategies are obtained for each subsystem, followed by a joint simulation, where the existing interconnections between the subsystems are represented, and the interchanges are defined according to pre-established rules.

One of the first experiences with this decomposition method is the "extended power pool" model, which was developed by the Norwegian Research Institute of Electricity Supply [11] [12] [13].

Although these references are recent, this model has been used for fifteen years in operation planning studies for the Norwegian system.

In this model, the operating strategies are obtained by a SDP algorithm that determines for each week of the planning period a vector with the expected marginal operating cost (water value). In the joint simulation phase, the interchange is defined in order to keep the subsystems at equal marginal costs.

In Brazil, some experiences with this approach were also developed. Reference [14] describes a procedure that uses a SDP model to determine the strategies, and a joint simulation model based on heuristic rules to define the interchange. Reference [15] presents a similar model that is presently used in operation planning studies. In Reference [16] a SDP model is also used to calculate the strategies. A matrix with the expected marginal cost of the system is obtained in a bi-dimensional state variables approach. The joint simulation aims to keep the systems at equal marginal costs.

This work presents a risk-constrained approach to find operating strategies of two interconnected systems, based on the decomposition method. The operating strategies are calculated for each subsystem by the risk-constrained SDP model previously described. The joint simulation is based on risk criteria and uses synthetic energy inflow series, obtained by the same stochastic model used in the determination of the strategies.

The energy interchanges resulting from the joint simulation are incorporated to the energy load of the subsystems, and new separate strategies are obtained, what starts the iterative process. Convergence is reached when the interchanges defined in two successive iterations are equal, within a certain range. The proposed procedure is illustrated in Figure 7.

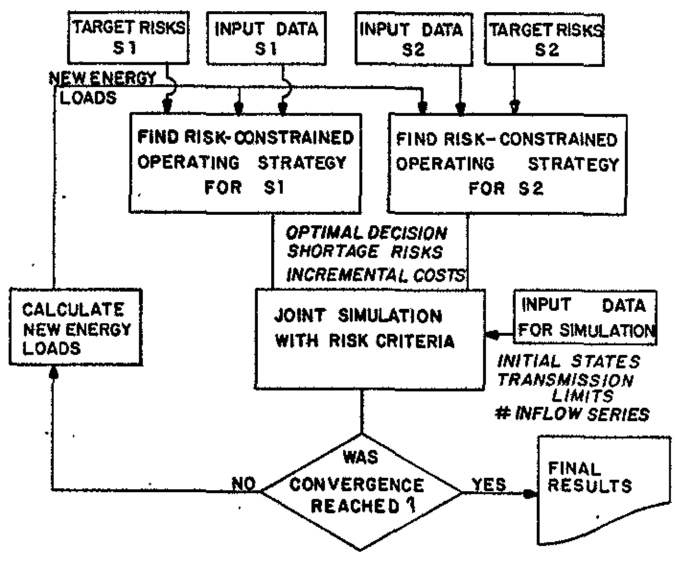

Figure 7 : Iterative procedure of the decomposition approach.

Risk criteria for the joint simulation

The joint simulation model uses risk criteria to define the energy interchange between the subsystems. At each month, for each inflow sequence, the initial states of the subsystems at the beginning of the month are known. Shortages risks from the operating strategies are associated to these states. The target risk of each subsystem in the stage is also known.

The energy interchange between the subsystems is defined according to the relative position of their shortage risk and target risk, as shown in Figure 8.

	$R^1 \leq TR^1$	$R^1 > TR^1$
$R^2 \leq TR^2$	A	B
$R^2 > TR^2$	C	D

Figure 8 : Relative positions of the subsystems.

In regions B and C of Figure 8, the subsystems are in an unbalanced situation. The system in the safe operation region will send energy to the other system, decreasing its stored energy until its shortage risk is equal to its target risk.

In region D, where both systems are violating their risk constraints, a possible decision could be not to interchange energy. However, this would make more sense for economically independent systems. In the Brazilian case, where the subsystems are operated under a global coordination, a better decision would be to keep the systems equally positioned with respect to their risk constraints, in a equi-risk operation.

For region A, where both systems are in the safe operation region, a possible criterion to define the energy interchange with an economical meaning would be to keep the systems at equal incremental costs, in a equi-cost operation. Another possible criterion would be to extend the equi-risk operation to region A, keeping the subsystems equally positioned above their risk constraints.

These two different criteria for region A led to the definition of two simulation procedures : equi-risk operation, which considers only the shortage risks, and equi-cost/equi-risk operation, which considers the incremental costs in the safe operation zone. Both options were implemented in the joint simulation model, and their results are compared in the next section.

Application

The iterative procedure and the simulation model were tested with the South and Southeast Brazilian subsystems, in the 1982-1986 period. The chosen initial states correspond to the discretized states (31,7) for the Southeast and (81,4) for the South in (101,10) discretization grids.

The same shortage risk, 3% in the planning period, was considered as the objective risk of the operation planning study for both systems. The natural risks estimated for the first stage were 18% for the Southeast and 11% for the South subsystem.

In the joint simulation, synthetic energy inflow sequences were independently generated for both systems, by the same auto-regressive model used to calculate the strategies. This assumption must be revised when applying this method to other systems.

The maximum interchange capacity values used represent the existing capacity at that time.

In the application of the decomposition method , sets of 100 sequences for each subsystem were initially used in the simulation. After convergence was reached, additional iterations were performed using sets of 1000 energy inflow sequences. With the equi-risk criterion, five initial iterations, plus two additional ones were necessary to obtain the strategies. With the equi-cost/equi-risk criterion seven initial iterations were performed, followed by two more, in order to reach the final convergence.

To illustrate the application of the iterative procedure, Figure 9 shows the monthly interchange values, obtained for the first two years of the planning period, in the five initial iterations with the equirisk criterion. Next, in Figure 10, the corresponding values obtained in the two additional iterations are shown.

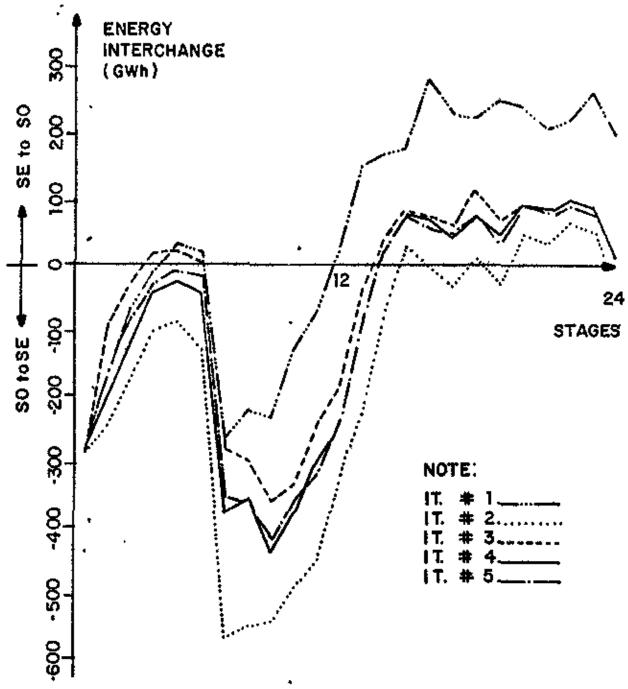

Figure 9 : Equi-risk simulation. Monthly interchange values obtained with sets of 100 sequences in the 1982-1983 period. Iterations 1 to 5.

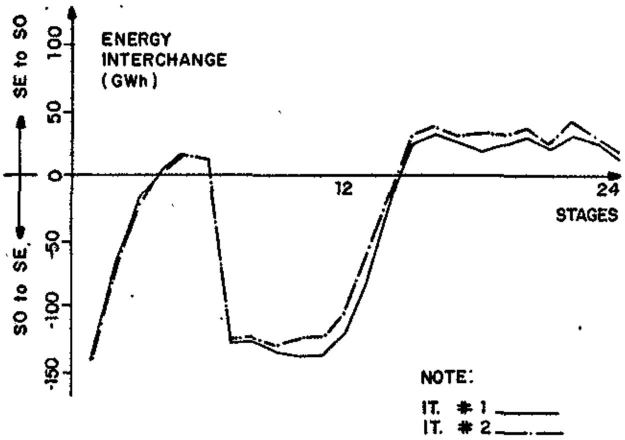

Figure 10 : Equi-risk simulation. Monthly interchange values obtained after initial convergence with sets of 1000 sequences in the 1982-83 period. Iterations 1 and 2.

Table I presents the results of the simulation with the final strategies, using sets of 1000 sequences, for both criteria. The results obtained considering the isolated operation of the subsystems are also shown.

It must be observed that, even though sets of 1000 energy inflow sequences were used in the simulations , from the statistical standpoint there is a great uncertainty on the real values of shortage risk of the systems. However, since the same sets of sequences were used in the three cases, it is possible to make some qualitative analysis of the operating criteria.

PARAMETER	SYSTEM	ISOL.	EQR.	EQC/EQR.
ESTIMATED SHORTAGE RISK	SE SO	4.1 0.4	1.6 0.1	1.7 0.1
THERMAL GENERATION (GWh)	SE SO TOTAL	278 289 567	262 272 534	250 277 527
THERMAL GEN. COST (CR$x10^6)	SE SO TOTAL	997 659 1656	899 625 1524	827 639 1466

Table I - Comparative analysis of the joint simulation criteria.

It can be noted in Table I that both methods are equivalent in terms of reducing the shortage risk in the period. In terms of reduction in the thermal generation cost in the period, the equi-cost/equi-risk criterion led to a higher economy, 11.5%, than the equi-risk criterion, 8% when comparing their results to the costs from the separate operation.

CONCLUSION

Operation planning models traditionally minimize the total operating cost, thermal generation cost plus energy shortages cost, of the system. However, since the economical effects of energy shortages are very difficult to quantify, what is done in practice is to use a penalty function to introduce a reliability constraint in the objective function. In this penalty-augmented approach, it is necessary to verify if the resulting strategy satisfies the reliability constraint. Different types of constraints, probabilistic or expected value constraints, may be used, but they do not lead to the solution of the real problem.

In fact, the operation planning problem is a set of nested problems for each stage of the planning period and this must be taken into account in the modelling.

A more realistic approach to obtain the operating strategies is to consider the minimization of the present expected thermal generation cost subject to a risk constraint that limits the shortage risk from each stage to the end of the planning period to previously fixed values.

A heuristic method for the definition of the risk constraint was developed. Although it has provided satisfactory results, other methods for the establishment of the target risks may be used.

The shortage risk surfaces of the strategies obtained by this risk-constrained SDP approach are characterized by presenting two distinct regions : a safe operation region, where the risk constraint is satisfied, and an unsafe operation region, in which the constraint is violated.

A decomposition approach was used to extend the study to the case of two interconnected systems, with satisfactory results. Two different criteria for the joint simulation of the subsystems were developed and tested for the South and Southeast Brazilian systems: equicost/equirisk and equirisk. Although they were equivalent in terms of reducing the expected shortage risk, the equicost/equirisk criterion provided a higher reduction on the expected thermal generation cost.

The following final remarks can be made about some aspects of this work.

In the implementation of the decomposition approach for the interconnected systems, the monthly average interchange resulting from the joint simulation was used to modify the strategies in the next iteration. The possibility of using more complete information should be examined.

One only value of objective risk was used for both systems in the case study. Some sensitivity analysis on the chosen value, as well as the implication of using the same value for the subsystems should be studied in the future.