COMPARISON OF STOCHASTIC AND DETERMINISTIC APPROACHES IN HYDROTHERMAL GENERATION SCHEDULING

T. de A. Araripe Neto
Departamento de Operação Energética, ELETROBRÁS
Rio de Janeiro, Brazil, P.O. Box 1639

C.B. Cotia[1]

M.V.F. Pereira[2]

J. Kelman
Centro de Pesquisas de Energia Elétrica, CEPEL
Rio de Janeiro, Brazil, P.O. Box 2754

Abstract. The objective of the optimal operation of a hydrothermal system is to determine a generation schedule that minimizes the operation cost along the planning period. The problem to be solved is stochastic and can be represented in a stochastic dynamic programming (SDP) framework. Since the computational requirements in this case tend to be heavy even for systems with a small number of reservoirs, deterministic equivalent methods are often adopted as an alternative to the SDP approach. These methods assume that the inflows are known along the planning period. The objective of this paper is to carry out a quantitative evaluation of the effect of using the stochastic representation of the inflows as compared with deterministic approaches in the generation scheduling of the Brazilian systems.

Keywords. Operation planning; Hydraulic systems; Dynamic programming; Stochastic systems; Modelling; Optimization.

INTRODUCTION

The use of hydroelectric resources in power production is still a relevant issue in some industrialized countries, such as Canada and Norway, and in the majority of the developing countries. In these countries, hydro generation contributes a large amount to the total energy production today, and, more significantly, represents the natural choice for system expansion, since there is still a large unexplored hydro potential (IAEA, 1984).

The Brazilian generating system is significantly dependent on hydro power and has grown very rapidly in size and complexity from the mid-sixties, as shown in Fig. 1.

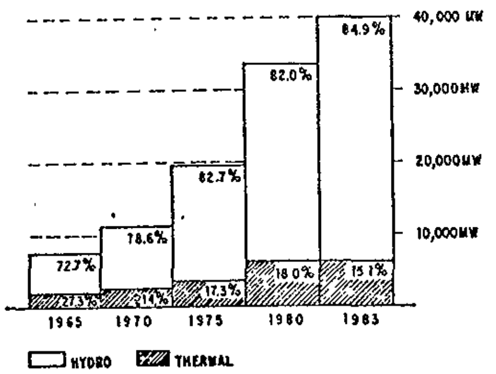

Fig. 1 - Installed Capacity of the Brazilian System

The share of hydro power on total energy production is even more substantial. In 1973, hydro generation was responsible for 89.4% of the total energy produced in the country (62,727 GWh). In 1983, this share increased to about 93.5%, for a total energy production of 161,970 GWh.

The operation planning problem in a hydrothermal system consists in determining a production schedule for the generating units that minimizes the generation cost along the planning period.

This problem is too complex to be solved by a single model, since it is a large-scale problem, with stochastic variables and non-linear equations (Pereira and Pinto, 1984a). Therefore, it is usually decomposed in a chain of subproblems, with different planning horizons and degrees of detail in the system representation. Results from each planning level are used as input to the next phase; feedback links are also used to help ensure a global optimization of the operation planning problem (Pereira and Pinto, 1984b).

The definition of the planning horizon and discretization steps, and the establishment of the most adequate system representation to be used in each planning level will depend on the characteristics of the system.

In the long-term planning level, an operating strategy is calculated to define the proportion of hydro and thermal generation that will be used to meet the load in each month of the planning period. This strategy should take into account the multi-annual evolution of reservoir storages, the risk of future energy shortages and the expected thermal generating cost.

The large inertia of the system reservoirs and the existence of hydrological homogeneity between the river basins of each region of the country allowed the use of an aggregate representation of the hydro system, known as "equivalent reservoir" model, to approximate the solution of the operation planning problem.

Operating strategies for the Brazilian interconnected systems were initially calculated by a deterministic approach which uses the aggregate re-

[1] Currently at Engevix S.A., Rio de Janeiro

[2] Currently on loan to the Electric Power Research Institute in Palo Alto, California

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presentation of the hydro system and is based on the historical inflow record. The system was operated according to a "rule-curve", which represents the minimum storage necessary to supply the load during the planning period under the worst inflow conditions of the past (GCOI, 1978).

In 1978, this approach was substituted for a stochastic dynamic programming (SDP) model, which had been jointly developed by ELETROBRAS, the Brazilian federal agency for electricity, and by CEPEL the Brazilian electric power research center, during the previous years (CEPEL/ELETROBRAS, 1977). The objective function of the SDP model is the minimization of the expected operation cost in the planning period, composed of thermal costs plus penalties for failure in load supply.

An alternative to the SDP approach is the deterministic equivalent method. This method assumes that the inflows are known along the planning period and determines a trajectory which will correspond to the optimal reservoir evolution for the pre-established inflow sequence. The solution is updated at each stage, as soon as new inflow forecasts become available.

The objective of this work is to carry out a quantitative evaluation of the effect of using the stochastic representation of the inflows as compared with deterministic approaches in the generation scheduling of the Brazilian systems. Therefore, operating strategies of the Brazilian South and Southeast aggregate systems are calculated by the three methods:

- rule curve
- deterministic equivalent
- stochastic dynamic programming

The resulting strategies are compared by simulating the system operation with a large number of synthetic streamflow sequences. The main performance measures are the risk of energy shortages, the expected thermal generation and the expected operation cost along the planning period.

THE RULE CURVE METHOD

From 1974 to 1978, the operating strategies of the Brazilian system were calculated by a deterministic approach, based on the historical inflows record.

This approach uses an aggregate representation of the hydro system. The set of existing plants and reservoirs is replaced by one equivalent reservoir and one run-of-river plant (Arvaniditis and Rosing, 1970). The energy storage capacity of this equivalent reservoir corresponds to the energy that can be produced by the complete depletion of all the reservoirs, for a given set of initial volumes and assuming a simplified operation rule. Similarly, the water inflow to the real system is represented by aggregate energy inflows, which are separated in controllable and run-of-river energy inflows.

The objective of the deterministic approach is to ensure load supply during the planning period under the occurrence of the worst inflow sequences of the past.

This criterion results in the determination of a rule-curve of the system, which indicates the minimum storage level in the beginning of each month, necessary to assure the energy load supply from that month to the end of the planning period, without having energy shortages, if any of the historical inflow sequences occurs.

The storage requirements for each sequence are calculated recursively, assuming that the thermal units are operated at their maximum rates, by the following expression:

$$L_{t,s} = \min(\bar{L}_t;\ \max(0;\ L_{t+1,s} - a_{t,s} - m_t - \bar{u}_t)) \qquad (1)$$

where

- $L_{t,s}$ is the minimum storage level at the beginning of month t, for inflow sequence s,
- s indexes the set of inflow sequences with the same lenght of the planning period that can be obtained from the historical record,
- $\bar{L}_t$ is the maximum storage level of the system in month t,
- $a_{t,s}$ is the energy inflow to the system in month t, in sequence s,
- m_t is the system energy load in month t,
- $\bar{u}_t$ is the maximum thermal generation of the system in month t.

The rule-curve value for each month, Z_t, is the maximum storage level from all the inflow sequences. Figure 2 illustrates the method

$$Z_t = \max_s \{L_{t,s}\} \qquad (2)$$

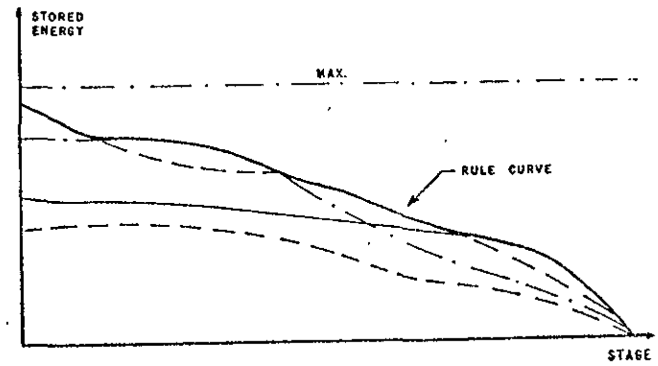

Fig. 2 - The Rule-Curve Calculation

In the system operation, if the storage level of the equivalent reservoir in the beginning of each month is above the rule-curve value, the thermal units are kept at their minimum rates during the month. If the rule-curve is reached, the thermal units are operated at maximum production.

This method was accepted and used in the planning activities because it was easy to implement and to understand. Operation planners intuitively felt that if the system was safe against the past, it would be safe for the future. However, this is not true since the historical record is one only outcome of the stochastic process of producing inflows. Additionally, it is not an economical way of operating the system, since the method has no minimization criterion of the operation cost.

These disadvantages were the basic reasons for the development of the stochastic dynamic programming model, which is described next.

THE STOCHASTIC DYNAMIC PROGRAMMING APPROACH

Since 1979 operating strategies for the Brazilian systems have been calculated by an optimization model, which uses a stochastic dynamic programming recursion, and in which the system is represented by the "equivalent reservoir" technique.

The objective function is to minimize the expected operation cost, composed of thermal generation costs plus penalties for failure in load supply. The state variables of the SDP model are the stored energy in the equivalent reservoir at the beginning of each stage and the hydrological trend of the system, represented by the total energy inflow during the previous stage. This last state variable is necessary because the inflows in successive stages are highly correlated. The decision variable in each state is the amount of thermal generation. The recursive equation can be written as:

$$f_t(x_t,a_{t-1}) = \underset{k\varepsilon K}{\mathrm{Min}}[CT(u_{tk}) + \underset{A_t|a_{t-1}}{E}\{\tfrac{1}{\beta}\, f_{t+1}\,(x_{t+1}(x_t,A_t,u_{tk}),A_t) + CD(d_t(x_t,A_t,u_{tk}))\}] \qquad (3)$$

where

$f_t(x_t,a_{t-1})$	is the present expected operating cost from stage t to the end of the planning period,
x_t	is the energy storage at the beginning of stage t,
a_{t-1}	is the aggregate energy inflow during stage t-1,
k	indexes the set of thermal generating decisions,
K	is the set of thermal generating decisions,
u_{tk}	is the k-th thermal decision at stage t,
CT(.)	is the thermal cost function,
$\underset{A_t\|a_{t-1}}{E(.)}$	is the expected value over the inflows, during stage t, A_t, conditioned by the inflow during stage t-1,
$1/\beta$	is the discount factor,
$x_{t+1}(.)$	is the system transition function,
CD(.)	is the deficit cost function,
$d_t(.)$	is the deficit function

Figure 3 illustrates the SDP recursion.

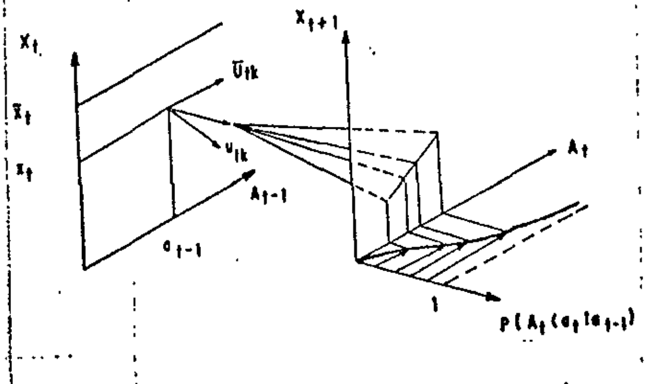

Fig. 3 - States Transition in the SDP Recursion

For a given state of the system in stage t, (x_t, a_{t-1}), each thermal decision of the set K is tested by the SDP algorithm. Since the probability distribution of the energy inflows in stage t conditioned by the inflow in the previous stage, $P(A_t|A_{t-1}=a_{t-1})$, is described by the stochastic model, it is possible to calculate the final states x_{t+1} by the transition equation. If the expected operation cost from each of these states in stage t+1 to the end of the planning period is known, and if they correspond to an optimal operation from t+1 to the final stage, the expected value of the total operating cost from (x_t, a_{t-1}) is calculated by equation (3). The thermal decision which results in the minimum operating cost will be the optimal decision for state (x_t, a_{t-1}), according to Bellman's Optimality Principle. The same procedure is repeated for each state and stage along the planning period.

It is necessary to extend the planning period in order to allow the use of a fixed value as terminal condition of the operating cost to initialize the recursion. Since the inflow is a stochastic variable, and a discount factor is used in the recursion to evaluate present costs, the optimal decisions for a certain period will be independent from the operating costs involved in a more distant horizon.

Typical discretization levels of the variables make the recursion very heavy in terms of computation time. In each stage of the planning horizon it would be necessary to use the transition equation 100,000 times to calculate the final states. Therefore, some procedures were developed in order to reduce the processing time of the optimization algorithm (CEPEL/ELETROBRAS, 1977). The search of the optimal decision is reduced in about 80% by these procedures.

The operating strategy thus calculated gives for each state in each stage the optimal thermal generation and the present expected operating cost. The incremental operating cost (water value) can be approximately obtained as:

$$f'_t(x_t,a_{t-1}) = \frac{\partial f_t(x_t,a_{t-1})}{\partial x_t} \cong \frac{f_t(x_t,a_{t-1}) - f_t(x'_t,a_{t-1})}{x_t - x'_t} \qquad (4)$$

Figure 4 shows the incremental cost surface.

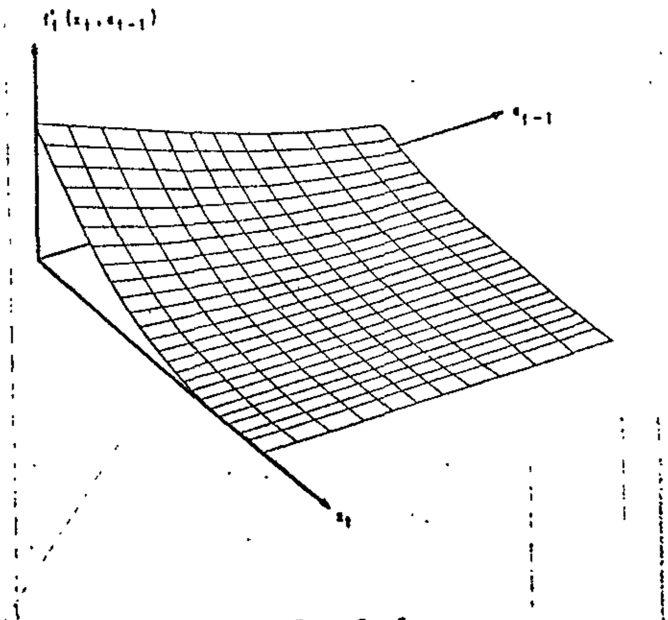

Fig. 4 - Incremental Cost Surface

The resulting strategy from the SDP model and consequently the energy supply reliability in the system operation are highly dependent on the penalty function CD(.) used in the objective function of the model. This function, which assigns a cost to the energy shortages, should ideally reflect the reduction in economical activities caused by the failure in load supply. This information should be obtained by complex macro-economical studies which depend upon updated and reliable statistical data about the technological relationship between the use of electricity and the final production in the country.

Since this information is not available, one possible approach to the solution of this problem is to use the deficit cost function as a parameter to ensure that the resulting operating strategy satisfies given reliability constraints. This approach uses an iterative procedure, in which the parameters of the penalty function are modified until certain supply reliability indexes are equal to pre-established values. These indexes are the shortage risk and the expected energy shortage in the period, which are estimated by simulating the system operation with a large sample of inflow sequences.

THE DETERMINISTIC EQUIVALENT APPROACH

The deterministic equivalent method is often used as an alternative for solving the generation scheduling of hydrothermal systems. Its basic characteristics are to ignore the inflows stochasticity and to retain the detailed representation of the generating system.

The assumption that the inflows are known along the planning period reduces the dimension of the scheduling problem, but even though it is still very complex. Several algorithms based on the deterministic equivalent approach have been developed in recent years (Hanscom and co-workers, 1980; Ikura and Gross, 1984; Murray and Yakowitz, 1979 ; Rosenthal, 1981).

The theoretical basis for using deterministic inflows is the certainty equivalence principle, which establishes that the optimal strategy for the solution of certain classes of stochastic control problems can be obtained by replacing the stochastic components by their expected value (Bryson and Ho, 1975).

The inflows in each stage are then substituted by forecasted values and the deterministic operation problem is solved. Since inflow forecasts are more accurate for shorter horizons, it is necessary to update the solution at each stage when new inflow forecasts become available. With the assumption that the best forecasting model can be represented in a stochastic dynamic programming framework, it is possible to compare the results of the SDP formulation and the deterministic equivalent approach in the operation scheduling of one single reservoir.

In this work, this single reservoir is the "equivalent reservoir" of the Brazilian South and Southeast regions and the inflows stochasticity is represented by a first-order autorregressive model. In this case, the only information that is needed to obtain the expected value of the inflow in the stage is the inflow that occurred in the previous stage. Since the inflow in the previous stage is one of the state variables of the SDP model, the information updating process of the deterministic equivalent method is completely reproduced in the SDP recursion.

Therefore, it is possible to obtain an operating strategy that reproduces the behavior of the deterministic equivalent method with the SDP model by replacing the conditional probability distribution of the energy inflow in each stage, which is supplied by the stochastic model, by an unit step located on the conditioned expected value, as shown in Figure 5.

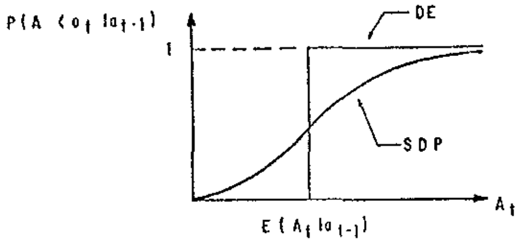

Fig. 5 - Representation of the Inflows

In theoretical terms, the hypothesis of deterministic equivalence is not well suited to the operation planning problem (Gjelsvik, 1981). Because the operation with deterministic inflows is more flexible than could be expected from the SDP solution, the deterministic equivalent methods will tend to use less thermal generation than the amount recommended by the stochastic solution (Read, 1979). However , a quantitative evaluation of this effect has been carried out for the systems of Turkey (Dagli and Miles, 1980) and New Zealand (Boshier and Read , 1981), with favorable results.

EFFECTS OF THE METHODS ON THE BRAZILIAN SYSTEM OPERATION

The three methods were applied to the South and Southeast Brazilian systems, for the 1985-1989 planning period. The resulting strategies are compared according to their expected operation cost in the five years horizon, which are estimated by simulating the system operation, from a given initial state, using a large sample of inflow sequences.

The expected operation cost is obtained as the sum of the expected thermal generation cost, and the cost of failure, which is the product of the expected energy shortage times the deficit cost function. In this application a linear cost function is used to penalize the energy shortages. It is worthwhile to remember that the cost function is not taken into account in the rule-curve method.

The simulations use sets of 1000 synthetic energy inflow sequences, produced by the same lag-one autorregressive model which is used in the SDP model.

Table 1 presents the main results of the simulation of the Southeast Brazilian system, with strategies calculated by the three methods.

The estimated shortage risk is defined as the number of inflow sequences with at least one monthly energy shortage during the planning period, divided by the total number of inflow sequences used in the simulation.

The resulting cost values are based on thermal generating costs of July/84 and on a deficit cost parameter of one million cruzeiros per MWh (.50US$/KWh).

TABLE 1 Comparison of the Methods for the Southeast Brazilian System from 1985 to 1989

PARAMETER	RULE-CURVE	DET. EQUIV.	SDP
Estimated Shortage Risk (%)	1.9	8.0	3.7
Expected Energy Shortage (GWh)	209	840	444
Expected Thermal Generation (GWh)	24,656	13,105	15,085
Expected Thermal Generation Cost (10^6US$)	413	71	115
Expected Total Operation Cost (10^6US$)	517	491	337

Table 2 presents the results of the simulation of the South Brazilian system.

TABLE 2 Comparison of the Methods for the South Brazilian System from 1985 to 1989

PARAMETER	RULE-CURVE	DET. EQUIV.	SDP
Estimated Shortage Risk (%)	5.5	12.4	10.1
Expected Energy Shortage (GWh)	104	269	207
Expected Thermal Generation (GWh)	28,634	19,930	20,300
Expected Thermal Generation Cost (10^6US$)	390	220	231
Expected Total Operation Cost (10^6US$)	442	354	335

The results indicate that for both systems, the rule-curve method produces the highest values of expected operating cost, what is an expected result. Since the rule-curve is calculated to assure load supply with the worst historical sequence, without having any minimization criteria, the resulting operation is characterized by a small energy shortage probability, with an intensive use of thermal generation.

The SDP approach produces the lowest values of expected total operation costs for both systems. The expected economy in the system operation when compared to the rule-curve method is about 34.8% for the Southeast system and 24.2% for the South system. These figures are an estimation of the benefits for the Brazilian systems, resulting from the use of the SDP model.

The results obtained by the deterministic equivalent method are situated in between the other two approaches. As it was expected, the deterministic equivalent method uses less thermal generation than the amount recommended by the SDP solution and consequently has higher energy shortages. This is due to the fact that the operation with deterministic inflows is more flexible than the resulting from the SDP approach. The expected total operating costs resulting from the deterministic equivalent approach are higher than the ones obtained by the SDP solution. For the Southeast system, the relative difference between the two methods is about 45.6%. For the South system this figure is about 5.8%.

The results obtained for the South system, with the expected total operating cost of the deterministic equivalent method quite close to the corresponding values of the SDP solution, are due to the fact that the reservoirs of this region have smaller inertia than the Southeast system, and therefore the system operation is less influenced by the stochastic component.

Figure 6 illustrates the differences between the three methods.

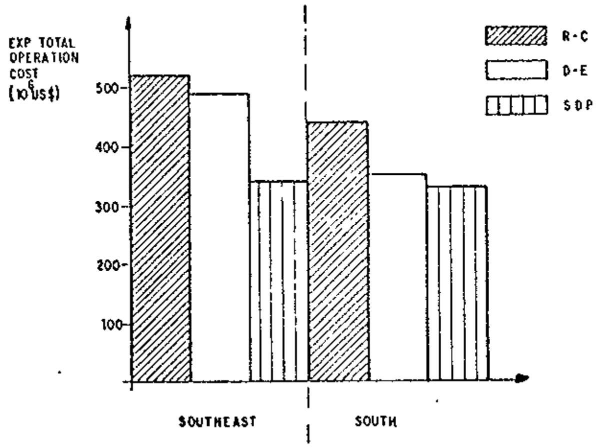

Fig. 6 - Expected Total Operation Costs Resulting from the three Methods

DISCUSSION OF THE RESULTS

The idea that deterministic equivalent approaches can be used for solving the generation scheduling problem of hydrothermal systems is usually justified by the fact that the information updating process of this method can replace the representation of inflow stochasticity.

This work does not constitute a complete comparison of the deterministic equivalent and the SDP approaches, since the possible benefits resulting from the detailed representation of the hydro system in the deterministic equivalent method are not considered. Additionally, this approach can benefit from being able to use more complex models to represent the actual stochastic process in the inflow forecasting. In the SDP approach, the use of such models would cause dimensionality problems.

The basic objective of this work is to evaluate the effect of using the first-order autorregressive model to represent the inflow stochasticity in the generation scheduling of the Brazilian aggregate systems.

The results correspond to a comparison of the deterministic equivalent and the SDP approaches for one single reservoir, with the assumption that the first-order autorregressive model is well suited for inflow forecasting.

As it was previously described, in this case the deterministic equivalent method can be seen as a particular case of the SDP formulation. It will be a good or bad approximation depending on the inertia of the system reservoirs and on the persistence embedded in the stochastic process.

For the Brazilian system, approximate schemes like the deterministic equivalent methods result in operating costs significantly higher than what is obtained by the SDP approach. Naturally, this is not a general result and the comparison of the methods depends on the characteristics of the system to which they are applied.

Although dimensionality problems do not allow the application of a SDP formulation for complex systems, an optimistic measure the possible gain in representing inflow uncertainty can be obtained by the following procedure.

1 - Evaluate the expected operation cost of the system by using the deterministic equivalent method with updating of the solution. Obtain C_1

2 - Calculate the optimal trajectory for one deterministic inflow sequence and evaluate the operation cost. Repeat this procedure for a large sample of inflow sequences and obtain the expected operation cost, C_2.

C_2 represents the lower bound of the expected operation cost of the system, since the perfect knowledge of the future inflows is assumed for each sequence. The difference between C_1 and C_2 will indicate the range of improvement that can be achieved by the representation of inflow uncertainty. If it is large, the stochastic component of the inflows should be considered in modelling the system. If it is small the hypothesis of deterministic equivalence is well suited to the system. As shown in Table 1, for the Southeast Brazilian system C_1 is equal to 491 million dollars. C_2 was evaluated as 114 million dollars according to step 2 of the procedure previously described. It should be noted that the expected operation cost of the SDP approach, 337 million dolars is in the (c_2, c_1) range, as expected.

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