FLOOD CONTROL IN A MULTIRESERVOIR SYSTEM

by

J. Kelman, Research Engineer
CEPEL-Electric Energy Research Center
J.P. da Costa, Research Engineer
CEPEL-Electric Energy Research Center
J.M. Damazio, Research Engineer
CEPEL-Electric Energy Research Center
C.P. 2754. Rio de Janeiro, Brazil
V.S. Barbalho, Civil Engineer
Engevix S.A., C.P. 1920, Rio de Janeiro, Brazil

ABSTRACT

It is presented a methodology for the design of flood con-trol multireservoir systems which uses linear programming. The methodology incorporates a set of deterministic restrictions on the flood control volumes derived by Mariën (1984) and a prob-ability of failure concept which is taken into account with the help of multisite daily synthetic streamflow sequences.

INTRODUCTION

A flood occurs in a river section P just below a reser-voir R_1 whenever the flow exceeds a critical value Q. An adaptation of Rippl's Method (1883) allows one to define a fea-sible region for the flood control volume K_1 sufficient to ensure the flow is always below Q, given an inflow sequence q(1), q(2), ..., q(h), as

$$K_1 \geq [\max_{t',t''} \{\sum_{t=t'}^{t''} (q(t) - Q)\},\ 1 \leq t' \leq t'' \leq h] \tag{1}$$

$$0 \leq K_1 \leq \overline{K}_1 \tag{2}$$

where $\overline{K}_1$ is a maximum volume.

Mariën (1984) generalized this result for the case when one can count with flood control volumes at several sites up-stream from R_1. Assuming instantaneous flow propagation, Mariën found a feasible region for the vector $\underset{\sim}{K}=[K_1, K_2, \ldots K_n]$ defined as

$$\sum_{j \in u} K_j \geq [\max_{t',t''} \{\sum_{t=t'}^{t''} ((\sum_{j \in u} q_j(t)) - Q)\},\ 1 \leq t' \leq t'' \leq h],$$

$$\forall u \in U \tag{3}$$

$$0 \leq K_j \leq \overline{K}_j \tag{4}$$

where,

j is the site index, j=1,...,n

q_j is the local inflow to site j (corresponds to the catchment

between site j and the immediately upstream sites);
u is a subset of $I_n = \{1, 2, \ldots, n\}$
U is the class of all subsets u which generate a "normal" partial reservoir system as defined by Mariño. A partial reservoir system is normal if and only if
a) the most downstream site (j=1) belongs to u
b) if $j \neq 1$ and $j \in u$ then there is $\ell \in u$ such that ℓ is immediatly downstream from j;
There are many constraints of type (3) as there are elements in U.

This paper expands this result in two ways. The first expansion concerns an objective function to be used together with the constraints defined by Mariño in order to find the optimal K^* vector. The second expansion concerns an algorithm to calculate the optimal K^* vector associated to a given probability of flooding at P. The greater is the probability the smaller will be the flood volume requirements. This algorithm considers a large number of multivariate synthetic daily streamflow sequences as the set of all possible inflow sequences.

METHODOLOGY

Consider a system of n reservoirs $R_1, \ldots R_n$ and a river section P downstream of all reservoirs at which the flow should not exceed with a given probability a critical value Q. Let S be set of all possible inflow sequences to the reservoirs.

Consider one of the constraints in (3). It is possible to find among the sequences belonging to the set S, a specific inflow sequence which will provide the greatest value for the right hand side of this constraint. Therefore all other sequences will provide redundant constraints.

The set of all non-redundant constraints in (3) are given by:

$$\sum_{j \in u} K_j \geq [\max_{s \in S} [\max_{t',t''} \{ \sum_{t=t'}^{t''} [(\sum_{j \in u} q_j^s(t)) - Q] \},\ 1 \leq t' \leq t'' \leq h]],$$

$$\forall u \in U \qquad (5)$$

Let's assume that the construction cost for each dam site is proportional to the flood control volume. The total construction cost is given by $\sum c_j K_j$. Therefore an optimization problem can be defined as:

$$\underset{\underset{\sim}{K}}{\text{minimize}}\ F(\underset{\sim}{K}) = \sum c_j K_j \qquad (6)$$

subject to

$$\sum_{j \in u} K_j \geq b_u, \ \forall u \in U \qquad (7)$$

where

$$b_u = \max_{s \in S} \left[\max_{t',t''} \left\{ \sum_{t=t'}^{t''} \left[\left(\sum_{j \in u} q_j^s(t) \right) - Q \right] \right\}, 1 \le t' \le t'' \le h \right] \quad (8)$$

and

$$0 \le K_j \le \overline{K}_j \quad (9)$$

The linear programming algorithm provides not only the optimal $\underset{\sim}{K}^*$ vector but also $\left.\frac{\partial F}{\partial b_u}\right|_{\underset{\sim}{K}^*}$. Each constraint in (7) is associated with a particular sequence $s \in S$. This constraint can be relaxed excluding s from S. In this way the old b_u value is replaced by the first ranked b_u value with S substituted by S/{s} in equation (8). A new application of the linear programming algorithm will provide another optimal $\underset{\sim}{K}^*$vector which will not be flood proof for the excluded sequence but will have smaller total cost. An estimate of the change of the minimal cost that will be achieved by excluding s is given by:

$$\Delta F_u = \left.\frac{\partial F}{\partial b_u}\right|_{\underset{\sim}{K}^*} \Delta b_u \quad (9)$$

The obvious choice is to exclude from S the sequence s that have the greatest ΔF_u.

The relaxation procedure above described is repeated as many times as required in order to get the solution associated with the given flood probability. For example, assume that there are 1000 sequences initially in S and the required return period for downstream flooding is 25 years,then 40 cycles would be necessary.

EXAMPLE

The algorithm was applied considering a system with eight reservoirs sites located in the Paraná Basin at southeast of Brazil (see Figure 1). The study was done with the help of 1000 "years" (October,1 to April,30) multivariate generated local daily flows (Kelman et al.,1985). It was selected a probability of flooding of 25^{-1} for an upper limit outflow from the system of 12000 m³/s. This flow at the outflow point of the system has in natural conditions a return period of 1.17 years. It was considered equal constructions costs coefficients ($c_j=1\ \forall j$). Table 1 shows the optimal flood control volumes obtained at each iteration of the procedure. Figure 2 shows the relation between the total flood control volume for the system (in this case assumed proportional to the total cost) and the probability of flooding.

Table 1. Optimal Flood Control Volumes at Each Iteration

Probability	I.Solteira	S.Simão	Itumbiara	Emborcação	A.Vermelha	Marimbondo	M.Moraes	Furnas	Total Volume
0.000	0.	166.	4128.	2436.	3645.	5260.	838.	10569.	27040.
0.001	0.	166.	2835.	2472.	1949.	4527.	917.	8319.	21186.
0.002	0.	166.	2694.	1298.	2532.	5260.	917.	7891.	20758.
0.003	0.	166.	2694.	1298.	2532.	5260.	917.	6081.	18947.
0.004	0.	166.	2694.	1298.	2532.	5260.	917.	6074.	18941.
0.005	0.	166.	2694.	1298.	2532.	5260.	917.	4759.	17625.
0.006	0.	166.	2694.	1298.	1953.	3721.	551.	7059.	17441.
0.007	0.	166.	2694.	1298.	1953.	3721.	551.	6179.	16561.
0.008	0.	166.	2694.	1298.	1953.	3721.	551.	5031.	15413.
0.009	0.	166.	2694.	1298.	1953.	3721.	551.	4624.	15006.
0.010	0.	166.	2694.	1298.	1953.	3721.	551.	4131.	14513.
0.011	0.	166.	2694.	1298.	1953.	3721.	551.	4081.	14463.
0.012	0.	166.	2694.	1298.	1953.	3721.	551.	4005.	14387.
0.013	0.	166.	2694.	1298.	1953.	3721.	551.	3780.	14163.
0.014	0.	166.	2694.	1298.	1953.	3721.	551.	3521.	13903.
0.015	0.	166.	2694.	1298.	1953.	3721.	551.	3257.	13639.
0.016	0.	166.	2694.	1298.	1953.	3721.	551.	2813.	13195.
0.017	0.	166.	2694.	1298.	1953.	3721.	551.	2741.	13123.
0.018	0.	166.	2694.	1298.	1953.	3721.	551.	2690.	13072.
0.019	0.	166.	2694.	1298.	1953.	3721.	551.	2530.	12913.
0.020	0.	166.	2276.	1319.	1692.	4379.	551.	2384.	12766.
0.021	0.	166.	2276.	1319.	1692.	4379.	551.	2370.	12753.
0.022	0.	166.	2276.	1319.	1692.	4379.	551.	2311.	12693.
0.023	0.	166.	2276.	1319.	1692.	4379.	551.	[illegible]	[illegible]
0.024	0.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.025	0.	[illegible]	2276.	1319.	1692.	4379.	551.	1316.	11698.
0.026	0.	166.	2276.	1319.	1692.	4379.	551.	1310.	11692.
0.027	0.	166.	2276.	1319.	1692.	4379.	551.	1301.	11683.
0.028	0.	166.	1323.	1003.	1244.	2597.	605.	4587.	11525.
0.029	0.	166.	1323.	1003.	1244.	2597.	605.	4557.	11495.
0.030	0.	166.	1323.	1003.	1244.	2597.	292.	4446.	11070.
0.031	0.	166.	1323.	1003.	1244.	2597.	292.	4191.	10816.
0.032	0.	166.	1323.	1003.	1244.	2597.	292.	4145.	10770.
0.033	0.	166.	1323.	1003.	1244.	2597.	292.	3897.	10522.
0.034	0.	166.	1323.	1003.	1244.	2597.	292.	3875.	10499.
0.035	0.	166.	1323.	1003.	1244.	2597.	292.	3865.	10490.
0.036	0.	166.	1323.	1003.	1244.	2597.	292.	3776.	10400.
0.037	0.	166.	1323.	1003.	1244.	2597.	292.	3698.	10323.
0.038	0.	166.	1323.	1003.	1244.	2597.	292.	3616.	10240.
0.039	0.	166.	1323.	1003.	1244.	2597.	292.	3522.	10146.
0.040	0.	166.	1323.	1003.	1244.	2597.	292.	3507.	10132.

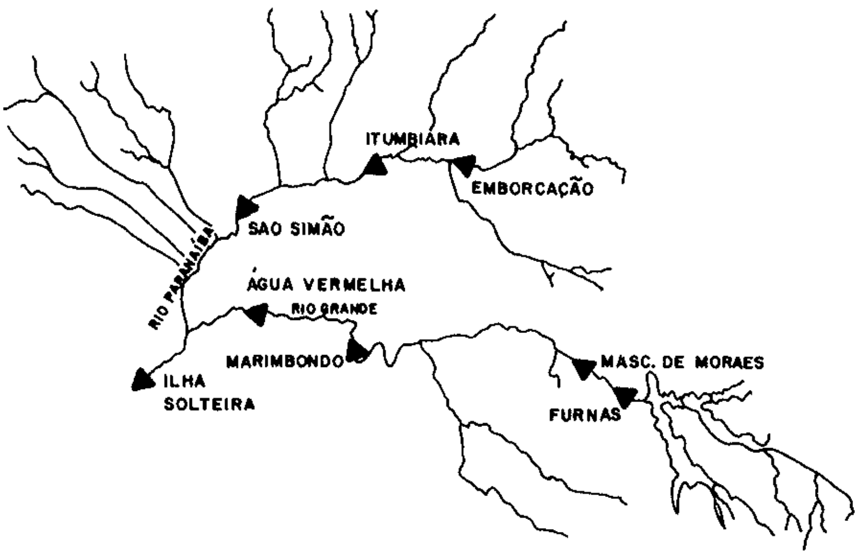

FIGURE 1-RESERVOIR SYSTEM

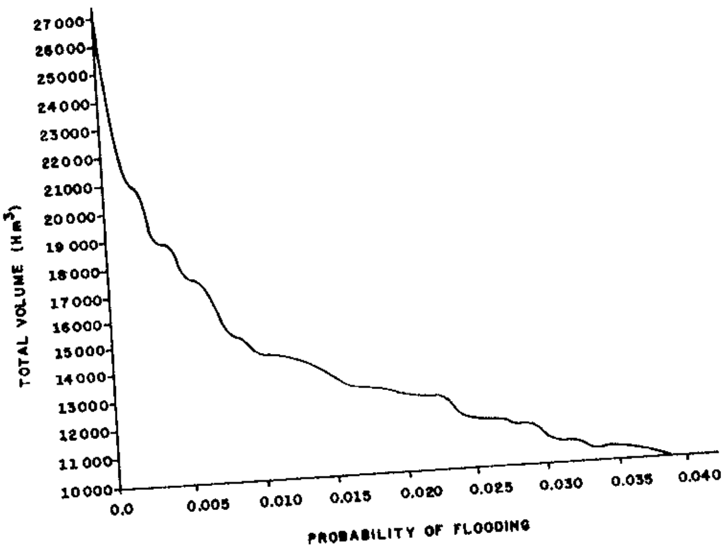

FIGURE 2. Relationship between flood control value and the probability of flooding

CONCLUSIONS

The present methodology is useful in the design of flood control multireservoir systems. The flood control volumes are optimally calculated in a linear programming framework which takes into account the probability of downstream flooding. Multisite daily synthetic streamflow sequences are an essential input to the methodology.

ACKNOWLEDGMENTS

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