Monte Carlo Approach to Spillway Design Floods

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ABSTRACT: This paper shows the use of a daily stochastic streamflow model for the calculation of the hydrograph for spillway design. Most of the literature related to the topic deals with the problem of how to evaluate the peak of the hydrograph with recurrence interval of T years, x(T). The usual approach is to fit a probability distribution F(.) to a set of m annual maxima $\underset{\sim}{x} = \{x_1, x_2, \ldots, x_m\}$ and get the estimate $\hat{x}(T)$.

It is well known that the smaller m, the smaller the precision of the estimate $\hat{x}(T)$. To be in the safe side, the engineer may design for the upper limit of a confidence interval around $\hat{x}(T)$, rather for $\hat{x}(T)$ itself. Confidence intervals may be calculated through parametric and non-parametric methods. This paper shows how a non-parametric method, the Bootstrap, can be used in two different ways to estimate the standard deviation of $\hat{x}(T)$:

a) re-sampling with replacement from the $\underset{\sim}{x}$ set
b) postulating a stochastic model for daily flows and re-sampling with replacement from the corresponding noise set

A demonstration is made with both alternatives, using an artificial process.

The availability of an extremely large number of synthetic daily streamflow sequences produced by a stochastic model is particularly helpful when one is designing jointly the spillway and the flood retention storage of a dam. In this case a T flood hydrograph is required rather than just the peak value. This paper describes how the Monte Carlo method was used to define the 10,000 year flood hydrograph (peak of 17344 m^3/s and 20 days volume of 21.3 $x10^9 m^3$) for the Serra da Mesa Dam, in the Tocantins River, Brazil.

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INTRODUCTION

There are several stochastic models for daily streamflow proposed in the literature which can be used in the selection of floods for spillway design. If there is no provision of retention storage the focus is on peak flows. In this case the classical approach aims to evaluate the peak flow with recurrence interval of T years, x(T), by fitting a probability distribution F(.) to a set of m annual maxima $\underset{\sim}{x} = \{ x_1, x_2, \ldots, x_m \}$. The main disadvantage of this approach is that usually the sample size (m) is small (as compared with T) so that x(T) is difficult to estimate accurately. In these situations one can get a better design by incorporating the uncertainty in the analysis using the Bayesian approach (Donald et al, 1972) or the Bootstrap method (Efron,1979). The latter has the advantage of being distribution free. The re-sampling technique of the Bootstrap can be done either directly with the set x or with a set of independent noises of a stochastic model.

When the spillway is associated with some retention storage the design requires the calculation of a full hydrograph, rather than just the peak value. In this case the availability of an extremely large number of synthetic daily streamflow sequences produced by a stochastic model is particularly helpful. The last section of this paper describes how the Monte Carlo method can be used in this situation.

BOOTSTRAP ESTIMATES

The bootstrap method requires m independent observations and proceeds by sampling with replacement from the original observations to obtain a new sample of size m, from which a statistic is computed. The process of sampling and computation is repeated a large number of times (say B) in order to obtain an estimate of the distribution of the statistic.

Direct Method

As usual for an annual maximum series, we treat $\underset{\sim}{x}$ as a set of mutually independent values. We sample from $\underset{\sim}{x}$ to yield new sets $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_B$, fit $F_i(.)$ to each $\underset{\sim}{x}_i$ and obtain the corresponding $\hat{x}_i(T)$.

The Bootstrap estimate of the standard deviation of $\hat{X}(T)$ is

$$STD^2 = \frac{1}{B}\sum_{i=1}^{B}(\hat{x}_i(T) - \hat{x}.(T))^2 \qquad (1)$$

where

$$x.(T) = \frac{1}{B}\sum_{i=1}^{B}\hat{x}_i(T) \qquad (2)$$

Use of Stochastic Daily Model

One may want to build a confidence interval for $\hat{x}(T)$ taking into account all the daily flows, rather than just the annual maxima. However the daily flows can not be considered independent, so it is necessary to employ a stochastic model to define a set of "noises" or "independent components" (Cover and Unny, 1986).

Consider the set of daily flows as a set of M "wet seasons" with N days each. With the MxN flows one builds a stochastic model and estimates a set of MxK noises (K < N). The re-sampling is done by the following algorithm:

a) re-sample with replacement the MxK noises
b) Separate the re-sampled noises in M sequences of K elements
c) For each of the M sequences use the stochastic model to generate a "wet season" of N daily flows and calculate its maximum.

Repeat these steps B times to obtain new sets $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_B$. Fit $F_i(.)$ to each x_i and compute the corresponding $\hat{x}_i(T)$. Equations (1) to (2) are used to estimate the standard deviation of $\hat{X}(T)$.

It should be noted that in the direct method only recorded maximum can be included in a re-sampled maximum series $\underset{\sim}{x}_i$. This is not true when the stochastic daily model is used, even if the daily flows are independent.

An Artificial Process

The simple AR(1) model was adopted to demonstrate the two approaches:

$$y_t = r\, y_{t-1} + \sqrt{1-r^2}\quad a_t \qquad (3)$$

where y_t is the flow on day t, r is the lag-one autocorrelation and a_t is the noise on day t, which is modelled as a standard normal deviate. No claim is made that equation (3) can be used to model actual daily flows. It is adopted here for the sake of simplicity. When it comes to real cases, more sophisticated models need to be used, such as the DIANA model (Kelman et al, 1985), adopted in the example of the next section.

Also, for simplicity we have considered N=30 "days". If r=0., the T-year flow can be easily calculated using tables of the normal distribution. In other cases a simulation can provide near-perfect estimates. Simulation can also be used to obtain standard deviations of x(T). Table 1 shows the values of x(100).

The experiment included 100 sequences of M=10 "wet seasons" apiece. Each of these sequences was re-sampled one thousand times (B=1000). $x_i(T)$ was obtained using the Gumbel distribution with the parameters estimated by the method of moments.

Table 1 compares the known standard deviations of x(100) for different values of r with the mean of the 100 Bootstrap estimates. It can be seen that the method that uses the stochastic model is more accurate, for this particular case. Both methods underestimate the true value.

	r=0.00	r=0.50	r=0.95
x(100)	3.40	3.39	3.07
std	0.48	0.56	0.76
std_1	0.37	0.40	0.52
std_2	0.39	0.47	0.64

Table 1: Summary of the experiment.
std_1-direct method, std_2 -use of stochastic model

MONTE CARLO APPROACH TO DAM-SAFETY ANALYSIS

Usually a proposed spillway is tested through a routing calculation with design inflow hydrographs, assuming the maximum normal water level (MNWL) of the associated reservoir as the initial condition for the reservoir storage. From these simulations one gets the maximum water level (MWL), to which we add allowances to wave run-up due to wind speed. These two levels are them compared with the dam crest level and account is given to possible hazards. The eventual underdesign (or overdesign) should be corrected by changing either the crest level, or the MNWL or the spillway capacity.

Alternatively one may test the proposed spillway by calculating the required MNWL through a backward routing calculation with the design inflow hydrograph, assuming the MWL corrected by wave run-up as a boundary condition.

Following the latter alternative, it is necessary to calculate the minimum attenuation storage (which might be zero) sufficient to prevent overtopping the MWL. For an inflow hydrograph j, the attenuation storage s(j), can be calculated as:

$$s(j) = \max_{t}[s_t(j)=\max[0 \; ; \; s_{t+1}(j)-y_t(j)+d \; [\; s_t(j),s_{t+1}(j)] \;] \qquad (4)$$

where:

t is the day index, $t=h,h-1,\ldots,1$

h is the last day of the flood season

j is the inflow hydrograph index

$s_h(j) = 0, \; \forall j$

$s_t(j) = 0$ implies that the water level is MWL

$y_t(j)$ is the inflow to the reservoir on day t

$d[s_t(j) \; ; \; s_{t+1}(j)]$ is the outflow from the reservoir through the spillway on day t

As the actual streamflow sequence is not known a priori, the attenuation storage is to be considered as a random variable. Its probability distribution can be inferred from a set $\{s(j), \; j=1,2,\ldots\}$ obtained from the use of equation (4) over thousands of synthetic sequences. The reliability of the design can be measured by the relative frequency of s(j) greater than the adopted value. This approach also can be easily used in the re-evaluation of operating constraints of existing dams (Kelman and Damazio,1983).

Table 2 shows the main characteristics of the Serra da Mesa project on the Tocantins River, central Brazil and data for testing a proposed spillway.

Total Reservoir Volume:	54.40 Km^3
Useable Volume:	43.25 Km^3
Drainage Area:	50,975 Km^2
Inundated area at MNWL:	1,784 Km^2
Installed Capacity:	1,200 Mw
Mean Inflow:	709 m^3/s
Proposed spillway characteristics	
MNWL:	460.00 m
MWL:	461.65 m
Attenuation storage:	2,91 Km^3
Number of gates: 5 (length of 15m, each)	

Table 2: Principal Characteristics of Serra da Mesa Dam and Reservoir.

In order to test the reliability of the proposed spillway 100000 synthetic "years" of 212 days each (October to April) were generated using the DIANA model (Kelman et al, 1985). Figure 1 shows a comparison between the sample accumulated distributions of maximum annual flow obtained from the historical information and from the 100,000 synthetic sequences. A comprehensive validation of these synthetic sequences is given by Damazio and Fuks (1985).

From the 100,000 synthetic sequences only 88 were considered for dam safety analysis. The adopted criteria was to select the hydrographs with peak flows greater than 15700 m^3/s which has a recurrence interval of 1000 years.

Table 3 gives the results. The required attenuation storage is only 0.98 Km^3, as compared to 2.91 Km^3, which is the value previously proposed for Serra da Mesa Dam. Because there may be some unexpected difficulties to operate the gates during a major flood, a sensitivity study was done assuming one of the gates to be inactive. In this case the attenuation storage should be 3.67 Km^3 and only 18 hydrographs out of 100,000 (recurrence interval of 5555 years) would possibly cause a dam break if the attenuation storage is selected as 2.91 Km^3, as originally planned. The 10th more adverse sequence, in terms of required attenuation storage, can be used as a 10,000-year flood.

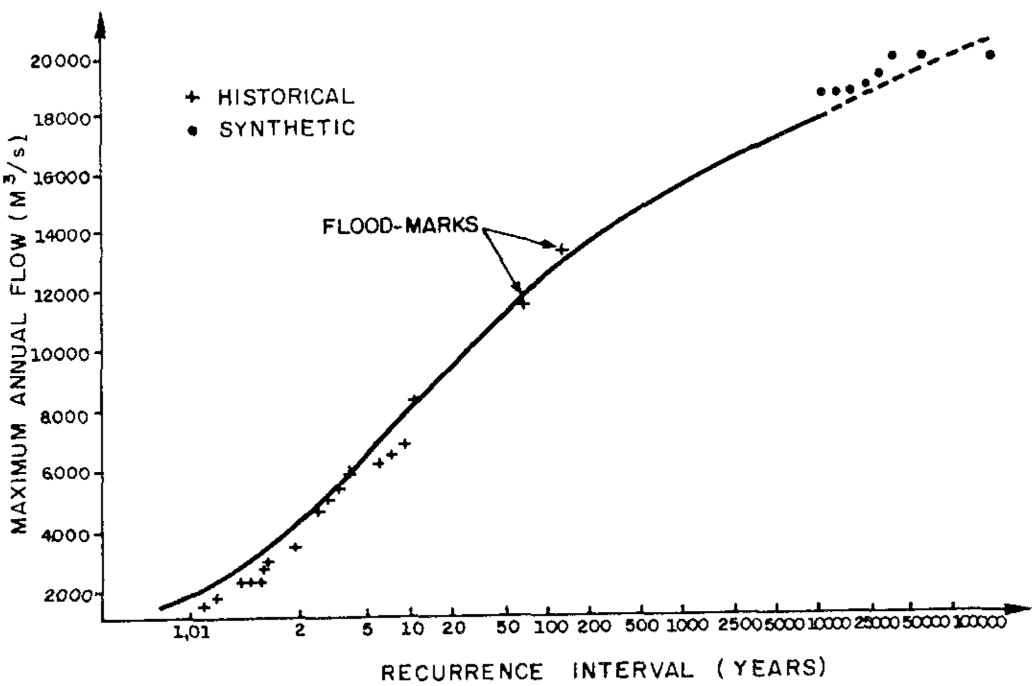

FIGURE 1 : ACUMULATED DISTRIBUTIONS OF ANNUAL MAXIMUM PEAK FLOWS AT SERRA DA MESA DAM, TOCANTINS RIVER, BRAZIL

Number of gates	Required attenuation storage for T=10000 (Km^3)	Risk for the originally proposed attenuation storage of 2.91 Km^3
5	0.98	none
4	3.67	$18x10^{-5}$

Table 3: Summary of the Monte-Carlo Study

CONCLUSIONS

Stochastic daily streamflow models can be very useful in the selection of floods for spillway design. Even if there is no space for retention storage the approach can be helpful in the determination of the peak flow with recurrence interval of T years, when the sample size is small as compared with T. When the spillway is associated with retention storage, the use of stochastic daily streamflow is a straightforward way to provide complete hydrographs.

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APPENDIX I. - REFERENCES

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