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The Determination of Flood Control Volumes in a Multireservoir System

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A methodology is presented for the optimal design of flood control volumes in a multireservoir system with one downstream critical section. The design is optimal in the sense that the objective function attempts to minimize the penalties associated with providing the flood protection. Moreover, the method explicitly considers a set of probability constraints on the occurrence of floods. The proposed calculation scheme is easily applied to almost any type of multireservoir system. The methodology is applied to the problem of determining the flood control volumes to be provided in a hydropower system of eight reservoirs on the Parana river in Brazil. In that case the objective function consists of minimizing the total firm energy loss.

INTRODUCTION

Sizing multireservoir systems is a difficult task due to the complex relationship between the decision variables (reservoir capacities) and the performance of the system. This performance is affected, for instance, by the stochastic nature of future inflows, the rules adopted for the system operation, complex cost and benefit functions, and noncommensurable objectives. This problem can be classified as a stochastic, multivariate, nonlinear, and multiobjective optimization problem.

Several optimization models based on different approximations for this problem have been proposed in the literature. An important family is the class of implicitly stochastic models, where the uncertainties in the inflows are taken into account by considering a large set of equally likely inflow sequences. The basic pitfall of this method is that the resulting size of the constraint set is proportional to the number of considered inflow sequences [*Stedinger et al.*, 1983]. Certainly in flood control problems the number of constraints becomes prohibitive due to the small time base one has to use.

Usual approaches to circunvent this difficulty are to use only the average inflow sequence [*Dorfman*, 1962] or the "critical periods" [*Hall et al.*, 1969] or yield models [*Loucks et al.*, 1981].

Mariën [1984] derived theoretical results concerning regulation of multireservoir flood control system and showed how these findings can be used to dramatically reduce the number of constraints of a stochastic model, without any kind of approximation. The results of Mariën are described in the ensuing text.

Assume a flood occurs in a river section P just downstream of a reservoir R_i whenever the flow exceeds a critical value Q. An adaptation of *Rippl*'s [1983] method allows one to define a feasible region for the flood control volume K_1 such that no flooding occurs for a given inflow sequence $q_1(1)$, $q_1(2)$, $\cdots$, $q_1(D)$ as

$$K_1 \geq \underset{1 \leq t' \leq t'' \leq D}{\text{maximum}} \sum_{t=t'}^{t=t''} (q_1(t) - Q) \qquad (1)$$

$$0 \leq K_1 \leq \bar{K}_1 \qquad (2)$$

where $\bar{K}_1$ is a maximum permitted flood control volume.

Mariën [1984] generalized this result for the case when there are several flood control volumes upstream of R_1. Assuming instantaneous flow propagation, Mariën found a corresponding feasible region for the vector $K = (K_1, K_2, \cdots, K_n)$ of flood control volumes of a so-called normal (see later on) n reservoir system given by the contraints

$$\sum_{j \in u} K_u \geq b_u = \underset{1 \leq t' \leq t'' \leq D}{\text{maximum}} \sum_{t=t'}^{t=t''} \left(\left(\sum_{j \in u} q_j(t) \right) - Q \right) \qquad (3)$$

$$\forall\, u \in U$$

$$0 \leq K_j \leq \bar{K}_j \qquad j = 1, \cdots, n \qquad (4)$$

where

- q_j local inflow in R_j (corresponding to the catchment between site j and the immediately upstream sites);
- u a subset of the set of integers $\{1, 2, \cdots, n\}$;
- U class of all subsets u such that the set of all R_i for $i \in u$ forms a so-called partial reservoir system (see later on).

According to the definitions in the work by *Mariën* [1984], a multireservoir system is normal if and only if for every reservoir R_i, except for R_1 (the most downstream site), there exists one and only one reservoir immediately downstream of R_i. Any set A of reservoirs R_i for $i \in u$ forms a partial reservoir system of the original system if R_1 belongs to A and if the reservoirs of A on their own form a normal reservoir system. For example, in Figure 1, taken from Mariën, the class U of

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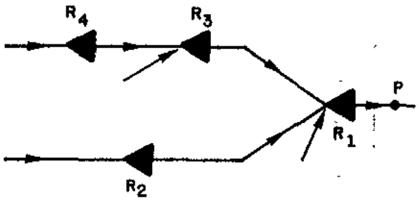

Fig. 1. An example of a four-reservoir system.

all subsets u consists of the subsets {1}, {1, 2}, {1, 3}, {1, 2, 3}, {1, 3, 4}, and {1, 2, 3, 4}, while examples of subsets not belonging to U are {1, 4}, {1, 2, 4}, {2, 3}, and {2, 3, 4}.

The constraints (3) form the so-called controllability conditions (CC). Clearly, there are as many CC as there are elements in U. *Mariën* [1984] shows that the CC can be easily calculated through the following recursive equation:

$$Z_u(t) = \max\left[0, Z_u(t-1) - Q + \sum_{j\in u} q_j(t)\right] \qquad t = 1, 2, \cdots, D \tag{5}$$

$$Z_u(0) = 0 \tag{6}$$

$$b_u = \max_t Z_u(t) \tag{7}$$

Mariën [1984] also shows how the CC can be easily generalized to systems with delayed instead of instantaneous flow propagation.

The model developed in this contribution adds two original features to the CC. The first element consists of using an objective function together with the CC to find the optimal K^* vector using linear programming. Second, an algorithm is given to calculate the optimal K^* vector associated with a given probability of flooding in P. This algorithm translates mathematically the trade-off between the value of the objective function and the flooding risk: a smaller return period of flooding (i.e., a larger flooding risk) corresponds to a better value of the objective function. As far as the authors are aware, this is the first approach to solve flood control design problems which allows to optimize system performance while taking into account the full details of the inflow process as well as probabilistic constraints on flood frequency.

Finally, the methodology is applied to multireservoir power systems. Whenever any of these reservoirs is prevented from filling completely due to flood control constraints, the firm power production of the system decreases. In predominantly hydroelectric systems, lack of water in the reservoirs during dry spells usually means energy shortage, rather than just higher generating costs. In this context, a new cost function related to firm power loss is derived and applied to the power system of the Parana river. This is a complex system of eight reservoirs partially in parallel and partially in tandem. Hence this example will illustrate the fact that the proposed methodology can be applied to almost any configuration of multireservoir systems. An earlier version of this methodology was presented at the Fourth International Hydrology Symposium [*Kelman et al.*, 1985*b*].

LINEAR PROGRAMMING FORMULATION

Consider again a normal n reservoir system $R_1, R_2, \cdots, R_n$, with a river section P, situated just downstream of the most downstream reservoir R_1. Flooding occurs whenever the flow exceeds a critical value Q. Let S be the set of all possible storms for which flood protection has to be provided. Each storm s consists of n inflow sequences $q_i^s(t)$ for $t = 1, \cdots, D$ and $i = 1, \cdots, n$, where D is the common duration of all the storms. In practice, D is the duration of the longest storm of the set S and one adds zero inflows to the data of the other storms to obtain a common duration for all the storms considered.

The right-hand side of constraint (3) for any storm $s \in S$ and for any $u \in U$ is by definition given by

$$b_u(s) = \underset{1\le t'\le t''\le D}{\text{maximum}} \sum_{t=t'}^{t=t''} \left(\left(\sum_{j\in u} q_j^s(t)\right) - Q\right) \tag{8}$$

which can be calculated using (5)–(7). With this notation the total set of CC can be written as

$$\sum_{j\in u} K_j \ge b_u(s) \qquad \forall\, s\in S \qquad \forall\, u\in U \tag{9}$$

However, for a given $u \in U$, it is possible to find among the storms $s \in S$ the one which will provide the largest value for the right-hand sides $b_u(s)$ for $s \in S$. Clearly, all other storms will generate redundant constraints. Hence the set of constraints (9) may be limited to

$$\sum_{j\in u} K_j \ge b_u[s_1(u)] \qquad \forall\, u\in U \tag{10}$$

where $s_1(u)$ is the storm s for which $b_u(s)$ is the largest. Note that, in general, different storms may create the largest $b_u(s)$ for different $u \in U$. Actually, each $s_1(u)$ is the first storm in the set of m storms of S, when those are written in a particular order associated with the partial reservoir system u. Each of these orderings

$$s_1(u), s_2(u), \cdots, s_m(u) \tag{11}$$

is defined by the relations

$$b_u[s_1(u)] \ge b_u[s_2(u)] \ge \cdots \ge b_u[s_m(u)] \tag{12}$$

provided that when $b_u(s')$ and $b_u(s'')$ are equal, the order of s' and s'' is arbitrary.

When one assumes that the costs for providing certain flood control volumes is proportional to these volumes, then a linear optimization problem (LP) can be formulated as

$$\underset{K}{\text{minimize}}\ F(K) = \sum_{j=1}^{j=n} \alpha_j K_j \tag{13}$$

subject to the constraints (4) and (10). The idea of replacing the constraints (9) by the constraints (10) makes solution of this LP feasible, even for a very large number of storms in S.

If needed one can replace (4) with a more general set of linear constraints which can, for example, represent the limitations on the flood storage provisions imposed by other purposes of the reservoir system. In the works by *Windsor* [1975, 1980] it is shown how such limitations can be formulated.

INTRODUCTION OF PROBABILITY CONSTRAINTS

The second problem considered in this paper consists of finding an optimal solution of (13) such that the resulting mean return period of flooding (i.e., when the flood control volumes are installed) is a given value of say, for example, T years. Suppose for a moment one has an historical record of y years of data, containing m storms, forming the storm set S. When only m' of these m storms do not fulfill the CC and when the number of years y is very large, then y/m' is a very good estimate of the mean return period of flooding. Hence to have a design of a mean return period of T years, one chooses

m' equal to the largest integer smaller than y/T. To have an optimal design of mean return period of T years, one solves the LP with objective function (13) subject to the constraints (4) and (9) where the set of storms S is replaced by the set $S^{m'}$, obtained from S by deleting m' storms in such a way that the reduction of the optimum value of the objective function $F(K^*)$ is maximized. The set $S^{m'}$ is obtained by constructing S^k from S^{k-1}, successively for $k = 1, \cdots, m'$, starting from S^0, which is the complete set S.

To construct S^1 starting from S, one solves the LP (13) with the constraint (4) and (9), with (9) replaced by (10) as explained before. The problem then is to find the storm $s \in S$ which is such that when (13), (4), and (9) are solved with S replaced by $S^1 = S/\{s\}$, the reduction of the optimal LP objective function value $F(K^*)$ is maximized. Since (9) is equivalent to (10), only the storm, belonging to the subset

$$C \equiv \{s_1(u) \text{ for all } u \in U\} \tag{14}$$

can provide binding constraints to the LP (13), (4), and (9) or (10). Hence one only needs to consider the storms of C as possible candidates for elimination. Moreover, when a storm $s \in C$ is eliminated, the change in objective function is given by

$$\delta F(s) = \sum_{u \in U(s)} \left.\frac{\partial F(K)}{\partial b_u(s)}\right|_{K^*} \{b_u[s_2(u)] - b_u[s]\} \tag{15}$$

where $U(s)$ is the subset of partial reservoir systems $u \in U$ which are such that $s_1(u) = s$, i.e., those $u \in U$ for which s is the storm which gives the highest values of $b_u(s)$, and $\partial F(K)/\partial b_u(s)|_{K^*}$ the partial derivative of $F(K)$ with respect to $b_u(s)$ at the point K^*, given by dual variable value for the constraint (10) associated with u from the LP solution.

The obvious choice is then to exclude from S the storm $s \in C$ which gives the most negative value for $\delta F(s)$. After obtaining S^1 as $S/\{s\}$ in this way, the ordering (11) for all $u \in U$ are adjusted to the fact that storm S is eliminated. To obtain S^2, the same procedure is repeated with S replaced by S^1 and S^1 replaced by S^2. After m' such iterations one obtains $S^{m'}$. Hence one only needs to consider the first $m' + 1$ elements of the orderings (11), since at most m' storms are eliminated for any given u.

With the proposed algorithm one needs to solve m' LPs, where the kth LP is defined by (13), (4), and (10), where $s_1(u)$ is the storm $s \in S^{k-1}$ for which $b_u(s)$ is the largest. The only difference between two successive LPs is that some right-hand sides of the constraints (10) are changed. In such a case it is recommendable to use the dual simplex method, since one can then obtain the solution of the next LP from the solution of the previous LP by only very few simplex iterations.

USE OF SYNTHETIC FLOW SEQUENCES

Normally, the long data sets required by the method proposed above are not available. Instead, one uses very long synthetically generated multisite flow sequences which have the same statistical properties as the real flows as far as storm frequencies and storm magnitudes are concerned.

The problem of selecting the storms in such a long synthetic data record can be easily solved by premising that when the previous storm has ended, the next storm s starts in the first period t_0 in which the total inflow given by

$$\sum_{i=1}^{n} q_i^s(t_0)$$

exceeds the critical flow Q. The same storm s ends in the first period t'' which is such that there exists a t' fulfilling the conditions

$$t_0 < t' \leq t'' \tag{16}$$

$$\sum_{i=1}^{n} q_i^s(t) \leq Q \qquad t = t', \cdots, t'' \tag{17}$$

$$\sum_{i=1}^{n} \bar{K}_i \leq \sum_{t=t''}^{t=t'''} \left(Q - \sum_{i=1}^{n} q_i^s(t)\right) \tag{18}$$

Clearly, this procedure for extracting the storms guarantees empty flood storage capacity at the end of each storm. It does not necessarily give the shortest possible storms, but this is of no importance here. Also, this procedure disregards any extra empty storage available in multiple purpose reservoirs at the beginning of each storm due to water consumption during the dry spell. In this case the return period of flooding can be actually greater than y/m'.

When in the studied river basin one can determine a definite flood season in each year, then the above storm selection procedure is not needed. Each simulated flood season then corresponds to just one storm, which extends from the first to the last period of the corresponding flood season. The return period of flooding is still given by y/m'. This corresponds to the classical formula that the mean return period is the inverse of the probability of flooding in one flood season, since this probability is estimated as m'/m, where m is equal to y in this case. Again, extra empty storage at the beginning of the flood season is not taken into account by the procedure and therefore the actual obtained return period can be greater than y/m'.

The right choice of y is not discussed here. It is a classical problem of experimental design in statistics.

The proposed methodology can be easily extended to the case were nonnegligible time delays exist for the waters flowing from one reservoir to another. The only adjustment consists of the right-hand side of (8), according to the directives given in the work by *Moriën* [1984].

A COST FUNCTION RELATED TO FLOOD VOLUMES IN HYDROPOWER SYSTEMS

The allocation of flood control volumes in the reservoirs of a power production system conflicts with the requirements of power generation: during the rainy season, flood protection is aided by low reservoirs levels and energy production benefits from full reservoirs. Hence it may be useful to calculate the flood volumes in such a way that the loss of firm energy is minimized, while keeping the flood risk below a preestablished level [*Kelman et al.*, 1982]. Firm energy can be defined as the maximum energy a hydro system can continuously provide if the worst historical drought happens again, given initial reservoir storage levels which account for flood storage.

During the flood season, there are always enough river flow to meet the firm energy requirement. However, if the reservoirs are not filled completely at the end of the flood season, due to the flood storage provisions, a decrease in firm energy may result later on, during the dry season. Hence one could define the cost function as the firm energy resulting from the simulation of the operation of the hydrosystem for the worst historical drought, with initial storages imposed by the flood storage provisions. For large systems with substantial carry over, the duration of the worst historical drought can last several years. In this case, the simulation will use storage provisions also as upperbounds for storage during subsequent flood seasons. In any case this would result in a firm energy given by $L(c_1 - K_1, c_2 - K_2, \cdots, c_n - K_n)$, where c_i is the capacity of reservoir i (only considering storages above minimum power pool) and K_i is that reservoir's flood storage allocation. For a given set of values $K_1, K_2, \cdots, K_n$ a value of L could be obtained through a simulation using the inflows of the worst historical drought.

The allocation of flood storage that gives the greatest value of firm energy is by no means obvious. For tandem systems there is a general trend to allocate flood storage in downstream reservoirs because water stored upstream will eventually flow to lower reservoirs. There are, however, some exception. Suppose for example that two in tandem reservoirs are very differently shaped: flood storage allocated to reservoir 1 (downstream) causes substantial loss of head, whereas practically no loss occurs in reservoir 2 (upstream). In this case, it can be a bad choice to allocate flood storage to the reservoir 1, because the minimum power production (firm energy) can decrease, at least during the beginning of the worst historical drought. Furthermore, there is no simple rule for parallel systems.

Because the construction of the objective function through successive simulations is quite cumbersome, it was decided to use the concept of system's energy [*Arvanitids and Rosing*, 1970; *Terry et al.*, 1986]. Suppose that the state of a n reservoir system at an instant t is defined as the vector $v(t) = (v_1(t), \cdots, v_n(t))$, where $v_i(t)$ is the stored water volume (above minimum power pool) in reservoir i at instant t. With some approximation, this state can be described by a univariate variable called the stored energy in the system and denoted by $g(v(t))$. This stored energy is given by the total energy that can be generated with this system in this state, supposing no future inflows and a reasonable rule for emptying the reservoirs. Assuming that there is a monotonic relationship between firm energy and the system's stored energy for all reservoirs at the flood control pool level, one can maximize the second quantity rather than the first one. Besides, maximizing the system's stored energy is a reasonable goal in itself in hydrothermal systems which are predominantly hydro because the thermal units are base loaded depending on the system's stored energy, as defined in the ensuing text [*Terry et al.*, 1986]. That is, creation of flood control storage results not only on generation expansion costs but also on operating costs.

An approximation for the stored energy is given by

$$g(c) = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j \left(\frac{h_j^{\max} + h_j^{\min}}{2} \right) c_i \qquad (19)$$

where $P_{ij} = 1$ for $j = i$; $P_{ij} = 1$ if j is a reservoir downstream from i; and 0 in all other cases; $h_j^{\max}$ is the head when $v_j(t) = c_j$; $h_j^{\min}$ is the head when $v_j(t) = 0$; and e_j is the efficiency of the jth power plant.

Note that as this approximation uses mean heads it is more related to situations where the relationship between head and storage can be approximated by a linear curve at least in the zone between $h_j^{\min}$ and $h_j^{\max}$.

In general, the stored energy at the end of the flood season, which is the quantity to be maximized, will be given by

$$g(c - K) = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j \left(\frac{h_j + h_j^{\min}}{2} \right) (c_i - K_i) \qquad (20)$$

with h_j the head in reservoir j when $v_j(t) = c_j - K_j$.

If it is assumed that h_j is given as a function of K_j in a linear way, which is consistent with the approximation used for the stored energy (equation (19)),

$$h_j = h_j^{\max} - a_j K_j \qquad (21)$$

then (20) can be written as (with $h_j^* = h_j^{\max} + h_j^{\min}$):

$$\begin{aligned} g(c-K) &= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j (h_j^{\max} - a_j K_j + h_j^{\min})(c_i - K_i) \\ &= \frac{1}{2} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j h_j^* c_i - \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j^* h_j^* K_i \right. \\ &\left. - \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j a_j K_j c_i + \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} e_j a_j K_j K_i \right) \end{aligned} \qquad (22)$$

The first term in the right-hand side of (22) can be omitted (it is a constant). After interchanging the indexes i and j and the summation sign of the third term of (22), the resulting objective function to be maximized is

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (-P_{ij} e_j h_j^* - a_i P_{ji} e_i c_j + P_{ij} e_j a_j K_j) K_i \qquad (23)$$

If h_j^* is sufficiently greater than $a_j K_j$, the nonlinear terms of (23) can be easily considered through an iterative optimization procedure, where in each iteration one minimizes a linear objective function representing the loss of stored energy, given by

$$\sum_{i=1}^{n} \alpha_i K_i \qquad (24)$$

with

$$\alpha_i = \frac{1}{2} \sum_{j=1}^{n} (P_{ij} e_j h_j^* + a_i P_{ji} e_i c_j - P_{ij} e_j a_j K_j^0) \qquad (25)$$

where the K_j^0, $j = 1, \cdots, n$ are given by the optimal solution of the previous iteration.

NUMERICAL EXAMPLE

The algorithm was applied to the problem of determining the flood control volumes to be provided during the flood season in a system of eight hydropower reservoirs of the Parana Basin (375,000 km^2) in the southeast of Brazil (Figure 2). In this study 1000 years were simulated ($y = m = 1000$) and the required return period of flooding was 25 years, so the number of storms to be eliminated is given by m' as

$$m' = y/T = 1000/25 = 40$$

The flows were obtained using a multivariate daily streamflow generator described elsewhere [see *Kelman et al.*, 1985*a*]. For this large river basin the use of a daily time base for flood studies is quite adequate. Whenever needed, the delays of the reservoir releases were taken into account.

The critical flow in P was 12,000 m^3/s. In natural conditions this flow has a return period of 1.17 years.

It is not conceivable that the flood control storage can be as large as the total reservoir capacity, since some water must be stored for peaking capacity. In this numerical exercise artificial limits for the flood storages $\bar{K}_i$ equal to 0.4 c_i were adopted. This will also guarantee that (21) well represents the variation of the heads. If this limitation was not adopted a nonlinear objective function would be necessary.

Table 1 shows important parameters of the reservoirs.

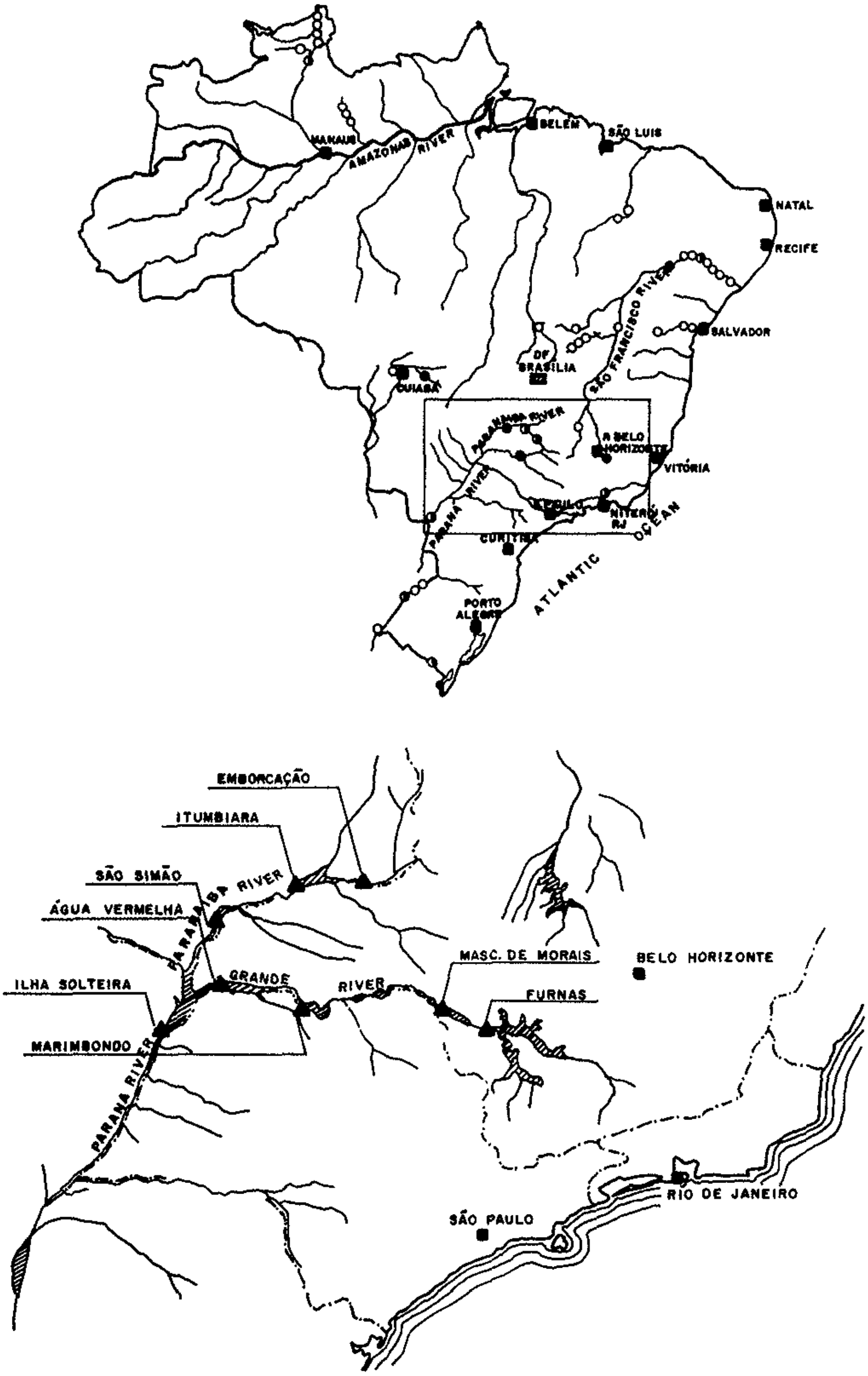

Fig. 2. The studied reservoir system.

Table 2 shows the optimal flood control volumes obtained at each iteration. At each iteration the algorithm given by (17) to adjust the nonlinearity of the objective function had to be used only once. Figure 3 shows the relation between the loss of energy (i.e., $g(c) - g(c - K)$) and the probability of flooding in one flood season (i.e., the inverse of the return period). The last row of Table 2 also contains the coefficients α_i of (25) for K_j^0 equal to zero (thse coefficients change at most 5% when the K_j^0 are equal to $\bar{K}_j$, there maximum value).

The successive solutions of Table 2 consist of putting a certain amount of total flood storage in those reservoirs which have the lowest coefficient α_i. This total flood storage is given by the CC (10) for u given by the set $\{1, 2, \cdots, 8\}$. For example, the last solution for a probability of flooding of 0.04

TABLE 1. Reservoirs' Parameters

	R_1, I. Solteira	R_2, S. Simão	R_3, Itumbiara	R_4, Emborcação	R_5, A. Vermelha	R_6, Marimbondo	R_7, M. Moraes	R_8, Furnas
c_i, 10^9 m^3	12.866	5.580	12.454	13.015	5.169	5.260	2.205	17.21
$\bar{K}_i$, 10^9 m^3	5.146	2.232	4.982	5.206	2.068	2.104	0.882	6.88
e_i	0.89	0.89	0.89	0.88	0.88	0.88	0.89	0.89
h_i^{max}, m	46.88	72.97	84.40	137.74	55.36	63.26	43.16	95.03
h_i^{min}, m	32.81	62.41	60.47	91.23	45.39	42.88	32.48	78.33
a_i, m/10^9 m^3	0.88	1.52	1.25	2.11	1.64	2.32	4.00	0.60

is simply given by providing the maximum possible amount of flood storage in the reservoirs R_1, R_2, and R_5, exactly those reservoirs which have the lowest α_i, and then an additional flood storage of 0.376×10^9 m^3 in R_6, the reservoir with the fourth lowest α_i.

This solution is simple because constraint (10) is binding only for u given by the set $\{1, 2, \cdots, 8\}$. However, this is not a general feature of the problem. To illustrate this, the example was altered artificially by interchanging all the data (c_i, $\bar{K}_i$, e_i, h_i^{max}, and h_i^{min}) of reservoir R_1 with those of reservoir R_7 (i.e., the capacity of R_1 becomes 2.205×10^9 m^3, while for R_7 it becomes 12.866×10^9 m^3, the h_i^{max} of R_1 becomes 43.16 m, etc.) while keeping the locations of the reservoirs and also the other data unchanged. Clearly, this will change all the α_i, whose new values are given at the second row of Table 3. This table also contains the solution of this altered system for a probability equal to zero. If these solutions were of the same simple type as for the original system then it would consist of providing as much flood storage as possible successively in the reservoirs R_5, R_2, R_6, R_1, R_7, R_3, R_8, and R_4 (i.e., in the order of increasing α_i). However, the solution of the altered system is not of this simple type. It provides some flood storage in R_4

TABLE 2. Optimal Flood Control Volumes at Each Iteration (10^9 m^3)

Probability	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
0.0000	5.146	2.232	4.982	2.365	2.068	2.104	0.882	6.887
0.0010	5.146	2.232	4.982	0.000	2.068	2.104	0.082	3.044
0.0020	5.146	2.232	4.982	0.000	2.068	2.104	0.882	2.904
0.0030	5.146	2.232	4.902	0.000	2.068	2.104	0.882	1.026
0.0040	5.146	2.232	4.982	0.000	2.068	2.104	0.862	0.902
0.0050	5.146	2.232	4.982	0.000	2.068	2.104	0.882	0.020
0.0060	5.146	2.232	4.982	0.000	2.068	2.104	0.634	0.000
0.0070	5.146	2.232	4.569	0.000	2.068	2.104	0.000	0.000
0.0080	5.146	2.232	3.654	0.000	2.068	2.104	0.000	0.000
0.0090	5.146	2.232	2.973	0.000	2.068	2.104	0.000	0.000
0.0100	5.146	2.232	2.604	0.000	2.068	2.104	0.000	0.000
0.0110	5.146	2.232	2.466	0.000	2.068	2.104	0.000	0.000
0.0120	5.146	2.232	2.412	0.000	2.068	2.104	0.000	0.000
0.0130	5.146	2.232	2.028	0.000	2.068	2.104	0.000	0.000
0.0140	5.146	2.232	1.939	0.000	2.068	2.104	0.000	0.000
0.0150	5.146	2.232	1.928	0.000	2.068	2.104	0.000	0.000
0.0160	5.146	2.232	1.487	0.000	2.068	2.104	0.000	0.000
0.0170	5.146	2.232	1.347	0.000	2.068	2.104	0.000	0.000
0.0180	5.146	2.232	1.187	0.000	2.068	2.104	0.000	0.000
0.0190	5.146	2.232	1.087	0.000	2.068	2.104	0.000	0.000
0.0200	5.146	2.232	1.044	0.000	2.068	2.104	0.000	0.000
0.0210	5.146	2.232	0.860	0.000	2.068	2.104	0.000	0.000
0.0220	5.146	2.232	0.765	0.000	2.068	2.104	0.000	0.000
0.0230	5.146	2.232	0.691	0.000	2.068	2.104	0.000	0.000
0.0240	5.146	2.232	0.170	0.000	2.068	2.104	0.000	0.000
0.0250	5.146	2.232	0.000	0.000	2.068	2.096	0.000	0.000
0.0260	5.146	2.232	0.000	0.000	2.068	1.987	0.000	0.000
0.0270	5.146	2.232	0.000	0.000	2.068	1.929	0.000	0.000
0.0280	5.146	2.232	0.000	0.000	2.068	1.866	0.000	0.000
0.0290	5.146	2.232	0.000	0.000	2.068	1.852	0.000	0.000
0.0300	5.146	2.232	0.000	0.000	2.068	1.290	0.000	0.000
0.0310	5.146	2.232	0.000	0.000	2.068	1.128	0.000	0.000
0.0320	5.146	2.232	0.000	0.000	2.068	0.909	0.000	0.000
0.0330	5.146	2.232	0.000	0.000	2.068	0.877	0.000	0.000
0.0340	5.146	2.232	0.000	0.000	2.068	0.857	0.000	0.000
0.0350	5.146	2.232	0.000	0.000	2.068	0.795	0.000	0.000
0.0360	5.146	2.232	0.000	0.000	2.068	0.663	0.000	0.000
0.0370	5.146	2.232	0.000	0.000	2.068	0.576	0.000	0.000
0.0380	5.146	2.232	0.000	0.000	2.068	0.544	0.000	0.000
0.0390	5.146	2.232	0.000	0.000	2.068	0.525	0.000	0.000
0.0400	5.146	2.232	0.000	0.000	2.068	0.376	0.000	0.000
α_i, kWh/m^3	0.17	0.32	0.47	0.74	0.27	0.41	0.53	0.66

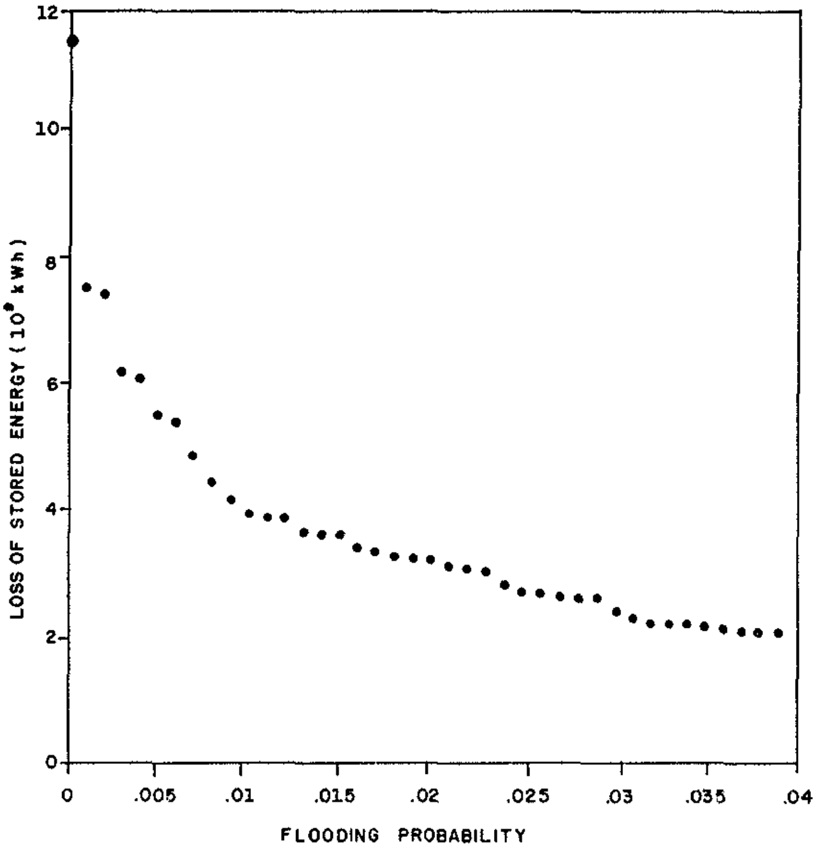

Fig. 3. The loss of stored energy as a function of the flooding probability.

without using all the possible flood storage in R_8, although α_8 is smaller than α_4. Examining this solution more closely, it was found that this was due to the limitations on the distribution of the total needed flood storage, imposed by CC (10) for u given by the set $\{1, 2, 3, 4, 5, 6\}$. This example shows how CC can play an important role in determining the allocated flood storages.

CONCLUSIONS

The presented methodology seems useful for designing or determining flood control volumes in a multireservoir system. It becomes possible to determine optimized volumes in an LP framework which takes into account the return period of downstream flooding. The availability of a model to generate synthetic multisite streamflow sequences with the emphasis on high flows is essential to the methodology.

The extension of the methodology to nonlinear objective functions seems possible. In the case of a linear objective function the dual simplex method is a special optimization routine which exploits the feature that the next optimization problem only differs from the previous one only by relaxing some contraints of (10). A possible subject of future research, is the construction and the use of such special routines in the case of nonlinear objective functions.

NOTATION

- a_j gradient of the head-storage relationship for reservoir R_j.
- b_u lower bound for the sum of flood control storages of reservoirs belonging to subset u (right-hand side of equation (3)).
- C set of the storms which provide the binding constraints.
- c_j capacity above minimum power pool of reservoir j.
- c vector of c_j, $j = 1, \cdots, n$.
- D duration of a inflow sequence.
- e_j efficiency of the jth power plant.
- F objective function.
- $g(v(t))$ stored energy in the system.
- h_j^{max} the head in reservoir R_j when $v_j(t) = c_j$.
- h_j^{min} the head in reservoir R_j when $v_j(t) = 0$.
- h_j^* $h_j^{max} + h_j^{min}$.
- i reservoir index.
- j reservoir index.
- K vector of flood control volumes.
- K_i flood control volume in reservoir R_i.
- $\bar{K}_i$ maximum permitted flood control storage volume in reservoir R_i.
- K^* optimal vector of flood control volumes.

TABLE 3. The Solution of the Altered System for a Flood Probability of Zero

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
K_i, 10^9 m^3	0.882	2.232	4.982	3.167	2.066	2.104	5.146	6.085
α_i, kWh/m^3	0.45	0.31	0.47	0.74	0.29	0.44	0.47	0.66

L firm energy.
m number of storms in S.
m' number of storms in S that do not fulfill the CC.
n number of reservoirs.
P river section just downstream of reservoir R_1.
P_{ij} indicator of topology.
Q critical value of flow in river section P.
$q_i(t)$ local inflow in R_i (corresponding to the catchment between site i and the immediately upstream sites) during period t.
$q_i^s(t)$ local inflow in R_i during period t for sequence s.
R_i ith reservoir.
S set of all possible storms.
S^k set of storms obtained from S by deleting k storms.
s sequence index.
$s_k(u)$ the storm s for which $b_u(s)$ is the kth largest.
T return period of flooding.
t time index.
U the class of all subsets u such that the set of all R_i for $i \in u$ form a so-called partial reservoir system.
u a subset of the set of integers $\{1, 2, \cdots, n\}$.
$v(t)$ vector of $v_i(t)$, $i = 1, \cdots, n$.
$v_i(t)$ stored water above minimum power pool in reservoir i at instant t.
y number of years corresponding to the storm set S.
α_j jth cost coefficient, corresponding to reservoir R_j.
$\delta F(s)$ change in the objective function when storm s is eliminated.

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