OPTIMIZATION OF FLOOD CONTROL AND POWER GENERATION REQUIREMENTS IN A MULTI-PURPOSE RESERVOIR

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Abstract. Dynamic Programming is applied to define the daily operating policy of a reservoir taking into account the inflow stochasticity. Since the use of Stochastic Dynamic Programming may involve many state variables in order to represent properly the daily flow process, an alternative approach, called Sampling Dynamic Programming is adopted. It allows the evaluation of mean costs of operation in each stage of the operating period, taking into account time dependence of streamflow by keeping track of the costs for each sequence of inflows separately. A study is done for the reservoir of Sobradinho, in São Francisco River, which is used for flood-control and energy-generation.

Keywords. Dynamic Programming; Flood Control; Optimization; Reservoir.

INTRODUCTION

The operation of a reservoir is often subject to conflicting requirements such as hydroelectric production and flood control. The optimization of power production requires the reservoirs to fill up during the rainy season in order to guarantee the load supply during the dry season. Flood control, on the other hand, requires free reservoir spaces to accommodate the incoming floods, thus minimizing downstream damage caused by excess outflow. In other words, any volume allocated for flood control reduces the energy production capability of the power plant and, vice-versa, any volume stored for power production reduces the degree of downstream protection.

The determination of the operation policy of such multipurpose reservoirs has to take into account two major problems:

a) It is difficult to find a common scale of measurements for both effects. A reduction in hydro production capability leads to an increase in thermal production and, consequently, in the system operating cost. Flood damage, on the other hand, may have associated monetary costs such as loss of property, ceasing profits, traffic deviation, etc., and non-monetary costs such as public health problems, social welfare, etc.

b) Because it is impossible to have perfect forecasts of the future inflow sequences, the problem to be solved is stochastic. Also, flood control decisions are made on a daily basis, and the stochastic process associated to daily inflow is rather more complex than those usually associated with monthly or annual inflows.

The objective of this paper is to describe the methodology developed to optimize the operation of a multipurpose reservoir. A composite objective function that expresses the preference of the decision-maker with regard to the distinc interests involved is defined and its impact over a range of possible values is investigated.

The solution technique developed is based on a dynamic programming recursion that is able to optimize the system operation over samples of daily streamflow sequences, either extracted from historical records or produced by synthetic streamflow models. Therefore, it is not necessary to assume a simple analytical model for the inflows, as required in the usual stochastic dynamic programming apporaches.

A case study with a system of hydroelectric plants in the lower São Francisco River (Northeast region of Brazil) is presented and discussed.

OPTIMIZATION OF THE OPERATION OF A RESERVOIR

The evolution in time of a reservoir is discretized in T stages t, t=1, ...,T. The state of the system is defined by a state vector x_t and can be, for instance, the storage s_t at each stage. Transition between stages is given by the continuity equation. A return function r_t is associated to each stage, and a composition of r_t, r_{t+1}, ..., r_T is used to evaluate the objective function f_t in stage t. This composition must satisfy the separability and monotonicity conditions (Yeh, 1982) for the Dynamic Programming (DP) equations to apply. Such is the case if f_t is taken as the sum, the product, or the maximum of r_t, r_{t+1}, ..., r_T. A decision d_t is then obtained to optimize the objective function f_t. As usual, it is assumed here that d_t is the target release for the reservoir. Also, the returns r_t are a function of the actual release u_t, as well as of the reservoirs storage at the end of the stage. If the inflows q_t are perfectly known, the actual release is always equal to the target release (u_t=d_t) and an optimal trajectory for the reservoir can be found, in the case of additive returns, by the recursive equation:

$$f_t(s_t)=\min_{d_t} \{r_t(u_t, s_{t+1}) + f_{t+1}(s_{t+1})\} \quad (1)$$

together with the continuity equation

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$$s_{t+1} = s_t + q_t - u_t \quad (2)$$

This approach is not possible, however, if the inflows that determine the reservoir's evolution are unknown. Instead, Stochastic Dynamic Programming (SDP) is used to obtain a policy that minimizes the expected value of the objective function. To take into account the persistence effects of streamflow, the state of the systems is often represented by (s_t, w_t), with $w_t = (q_{t-1}, q_{t-2}, \ldots, q_{t-\ell})$ being a vector of inflows in the previous periods. In this case, the equation that yields the optimal policy is

$$f_t(s_t, w_t) = \underset{d_t}{\mathrm{Min}} \; \underset{q_t|w_t}{E} \; [\, r_t(s_t, u_t) + f_{t+1}(s_{t+1}, w_{t+1})] \quad (3)$$

where, for a target release d and an inflow q_t, u_t is given by

$$u_t = \mathrm{Min}\{s_t + q_t - s_{min}), \mathrm{Max}(s_t + q_t - s_{max}), d\}\} \quad (4)$$

In the equations above, $\underset{q_t|w_t}{E}[\cdot]$ represents the conditional expectation of q_t given w_t, and s_{min} s_{max} are respectively the lower and upper bounds of the sotrage.

It will be necessary, therefore, in pratical applications of SDP, to estimate the conditional probability distribution of the inflow q_t, given $q_{t-1}, q_{t-2}, \ldots, q_{t-\ell}$.

SAMPLING DYNAMIC PROGRAMMING (SADP)

When using the inflows in previous periods as state variables, one is assuming that the inflow process is (in fact) autoregressive of order ℓ. Indeed, if this is the case the vector w_t is all the relevant information needed by the optimization procedure given by the equation (3). It is common practice to use (s_t, q_{t-1}) as state variables to obtain yearly or monthly, operating policies. However, to solve the problem of daily operation of a reservoir, considering only q_{t-1} may not be enough. On the other hand, addition of past inflows of higher orders as state variables increases the computational effort considerable, the so-called "curse of dimensionality". Besides, it requires estimating the many parameters of the conditional probability distribution of the inflows. These problens can be avoided by using a sampling approach rather than an analytical expression for the conditional probability distribution. This approach here called Sampling Dynamic Programming, was used by Araujo and Terry (1974) for the operation of a hydrothermal system. Its use for the daily operation of a reservoir is described in the following.

Streamflow data is kept chronologically ordered, yielding for each year of record one realization of the corresponding stochastic process. These streamflow sequences are used to simulate the reservoir's evolution in all possible combinations of storage and inflow sequences, for each stage, in a backward procedure that uses DP equation for the calculation of an optimal policy.

Let

$s_t(k)$ be the reservoir's storage at stage t, discretized in N states (k=1,...,N) with $s_t(1)$ = empty and $s_t(N)$ = full;

$q_t(i)$ be inflow at stage t, for the ith sequence (i=1,...,M), M being the number of years of streamflow record;

$d_t(k)$ be the target release in state k, stage t ;

u be the actual release at any stage or state;

$r_t(s_{t+1}, u)$ be the return at stage t, r_t may be regarded as the cost of a release u and a reservoir level corresponding to s_{t+1} at the end of the stage;

$f_t(s_t, i)$ be the cost of the operation of the reservoir from state t to the end of the planning horizon, when the storage is s_t and the ith sequence of inflows is occurring.

For every state and stage, the target release $d_t(k)$ is given by

$$\underset{d}{\mathrm{Min}} \; \{(1/M) \sum_{i=1}^{M} (r_t(s_{t+1}, u_t) + f_{t+1}(s_{t+1}, i)\} \quad (5)$$

where for each d, u_t and s_{t+1} are given by (4) and (2), with $q_t = q_t(i)$ and $s_t = s_t(k)$. Once $d_t(k)$ is found, the cost functions f are updated separately for each sequence.

$$f_t(s_t(k), i) = r_t(s_{t+1}, u) + f_{t+1}(s_{t+1}, i) \quad i=1,\ldots,M \quad (6)$$

Again, u_t and s_{t+1} in equation (6) are given by (4) and (2), with $d = d_t(k)$.

That procedure, repeated for t= T,T-1,...1 and K = 1, ..., N , yields an operating policy for the reservoir close to the optimal, in the sense that it minimizes its estimated mean cost of operation.

It can be seen that in equation (5) the expected cost of operation is estimated from sample values for all the M sequences of inflows, whereas in equation (3) an analytical expression is used.

Garabedian and Meslier (1979) also adopted sample values to estimate the expected cost of operation, for the French hydrothermal system. They used the mean value of f_{t+1} (s_t) along the sequences to estimate $f_t(s_t)$, which is equivalent to optimize the system which only one state variable, the storage, using the sample marginal distribution of f_t. In SADP, on the other hand, the cost of operation is calculated conditioned on the sequence of inflows, and only then averaged over all sequences.

FLOOD CONTROL AND ENERGY GENERATION IN THE SÃO FRANCISCO RIVER

Case Study

The technique of SADP was applied to a system of hydroelectric plants in the lower São Francisco River, where an attempt was made to obtain an operating policy for flood control coupled with energy-generation. Since these two objectives are conflicting, an optimal policy should minimize the aggregate cost due to flooding and energetic operation of the system.

The system of hydroelectric plants studied is composed of three reservoirs: Sobradinho, Moxotó and Paulo Afonso. The main regulating capacity is due to the upstream reservoir of Sobradinho and the whole system can be optimized through its operating policy. Sobradinho's main data is given in Table 1.

Hydrology

There are 52 years of flow recorded at the city of Juazeiro, approximately 40 km downstream of the Sobradinho reservoir, from 1929-30 to 1980-81 . This record, considered to be of good quality, was used for the design of the reservoir. A period of 212 days, from October 1st to April 30th, which comprises the flood season for that region, was extracted from each hydrological year. Mean standard deviation, coefficient of skewness and coefficient of kurtosis are shown in Table 2 for these hydrological sequences and for the annual maximum flows. Fig. 1 shows the empirical cumulative distribution function of the annual maximum flow.

Table 1. Characteristics of the Sobradinho Reservoir

Regulated flow	2060 m³/s
Long-term average flow	2800 m³/s
Maximum outflow capacity	22,835 m³/s
Total reservoir surface	4241 km²
Maximum volume	38.541 km³
Maximum normal volume	34.116 km³
Minimum volume	5.477 km³
Maximum water level	393.50 m
Maximum normal water level	392.50 m
Minimum water level	380.50 m

Table 2. Statistics of the Sequences of flows in Juazeiro during the flood season

	Average (m³/s)	St.Deviation (m³/s)	Skewness	Kurtosis
Daily flow	3523	2196	1.36	6.72
Maximum Annual Flow	6996	2708	2.13	7.90

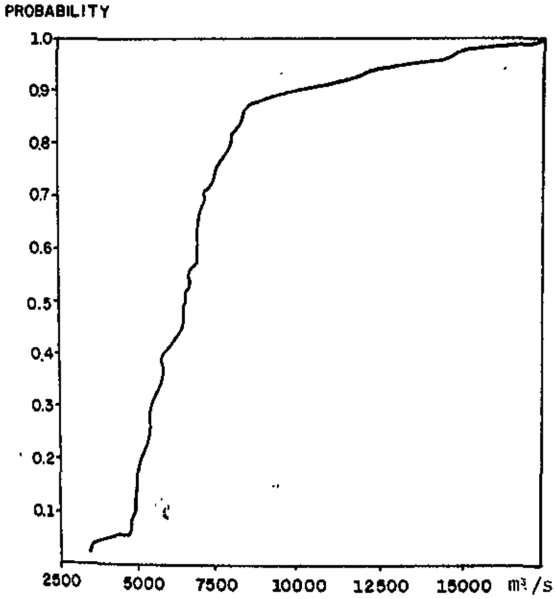

Fig. 1. Empirical Cumulative Distribution Function of the Annual Maximum Flow in the Reservoir of Sobradinho

Energy Generation

Energy generation costs are incurred whenever the system fails to meet the demand, which is assumed in this study to be equal to the firm power, 2282 MW. This can happen either during or after the flood season. During the flood season, the energy cost is considered to be a linear function of the actual release u, with u=2000 m³/s being the minimum release that meets the energy demand under a mean head of 137 m. In this case, this function, $r_0(u)$, is

$$r_0(u) = \begin{cases} 0, & u \geq 2000 \text{ m}^3/\text{s} \\ b_0\,(2000 - u), & u < 2000 \text{ m}^3/\text{s} \end{cases} \quad (7)$$

The expected cost due to energy-generation deficits from the end of the flood season until a planning horizon 5 years ahead is taken as function of the storage in the last day of the flood season. It can be found by applying Stochastic Dynamic Programming to evaluate the monthly operating policy for the hydrothermal system (Araripe, Pereira and Kelman, 1984). The Stochastic Dynamic Programming procedure also gives the marginal cost of the energy deficit in US$/MW from which the value of b_0 in eq. (7) can be found by a proper transformation of release u into power, keeping the assumption of constant mean head.

Flood Control

In the years of 1979 and 1980 there were large floods in the lower São prancisco River. The occurrence of these floods led the authorities to consider the necessity of flood control in the river. The Electricity Company of the São Francisco - CHESF - decided to allocate part of the useful storage of the Sobradinho reservoir to prevent flooding of the cities of Juazeiro and Petrolina, downstream. The cities at the margins of the reservoir's lake should also be protected. This means that the release from the reservoir should be limited to 6000 m³/s, and the water level in the reservoir should not exceed the maximum normal water level lest flood damages occur. Two cost functions, r_1 and r_2, are employed to represent these constraints. They are defined by

$$r_1(u) = \begin{cases} 0, & u \leq 6000 \text{ m}^3/\text{s} \\ b_1\,(u - 6000), & u > 6000 \text{ m}^3/\text{s} \end{cases} \quad (8)$$

$$r_2(s) = \begin{cases} 0, & h(s) \leq \text{MNWL} \\ b_2\,(h(s) - \text{MNWL}), & h(s) > \text{MNWL} \end{cases} \quad (9)$$

Where h (.) is the elevation-storage relation for the Sobradinho reservoir.

Objective Function

Once the functions $r_0(.)$, $r_1(.)$ and $r_2(.)$ are defined, it is convenient to store the costs due to these three different causes sepparately. For each sequence i and stage t, therefore, there will be three functions, $f_{0t}(s,i)$, $f_{1t}(s,i)$, $f_{2t}(s,i)$, representing these three types of cost from stage t on. Flood costs are considered to be zero after the flood season. This is represented by

$$f_{1,T+1}\,(s,i) = 0 \text{ all } i \quad (10)$$

$$f_{2,T+1}\,(s,i) = 0 \text{ all } i \quad (11)$$

As mentioned previously, the costs of deficit after the flood season are given by the function g (s). For all the sequences, $f_{0,T+1}(s,i)$ is represented by that function.

It is interesting to consider how costs should be calculated once the functions r_0, r_1 and r_2 are defined. Deficit costs are additive, since they represent interruptions in the industrial production and commercial activities. It is not clear, however, how to compute flood costs. One way is to establish that flood costs are a function of the maximum level reached by the waters during the flood season. The recursive equation of SADP for this case is given by Eq. (12).

On the other hand, if it is considered that floods cause interruption in commerce and economical activities whose costs would also be additive, a second objective function would be proposed as Eq. (13).

$$\min_{d} \{(1/M) \sum_{i=1}^{M} [r_{o,t}(u) + f_{0,t+1}(s_{t+1},i) + \max\{r_{1,t}(u), f_{1,t+1}(s_{t+1},i)\} + \max\{r_{2,t}(s_{t+1}), f_{2,t+1}(s_{t+1},i)\}]\} \quad (12)$$

$$\min \{(1/M) \sum_{i=1}^{M} [r_{o,t}(u) + f_{o,t+1}(s_{t+1},i) + r_{1,t}(u) + f_{1,t+1}(s_{t+1},i) + r_{2,t}(s_{t+1}) + f_{2,t+1}(s_{t+1},i)] \quad (13)$$

A second problem is to evaluate these costs, that is, to assign values to b_1 and b_2. Along with the difficultyof evaluating the material damages of a flood, there is also a "social cost" involved due to the many disturbances that flooding causes to the citizen's lives. Ultimately,the cost that the society is willing to pay to prevent flooding, or the risks that is prepared to take, will be a matter of political decision. In order to avoid these problems which do not concern directly the technique of SADP, a sensitivity analysis was made for the two objective functions presented above. As mentioned previously, the value of b_0 comes from the SDP for the monthly operation of the system, being an accepted value adopted in the planning of the operation. For the system studied its value is 50,00 US$/(m³/s).

RESULTS

For the objective function given by Eq. (13) additive flooding costs, four cases were studied, with values for b_1 and b_2 given in Table (3).

It was found that in cases 1 and 2 the operating policy is very insensitive to floodings, being concerned chiefly with energy deficits. The operating policies obtained in cases 3 and 4 are respectively given in Figs. 2 and 3. The lines define regions where the target release d is less than or equal to some value d*, with d* = 2000, 4000, 6000, 8000, 10000 m³/s.

Table 3. Marginal Flooding Costs Studied for SADP

	Case 1	Case 2	Case 3	Case 4
b_1(US$/(m³/s)	42.86	428.6	4,286	42,860
b_2(10^6US$/m)	0.150	1.50	15	150

For the objective function fiven by Eq. (12) the operating policy obtained didn't lead to significant flood control operation, except in case 4.

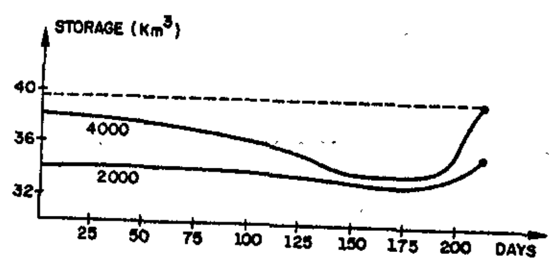

Fig. 2. Daily Operating Policy for Sobradinho Additive Flooding Costs, Case 3

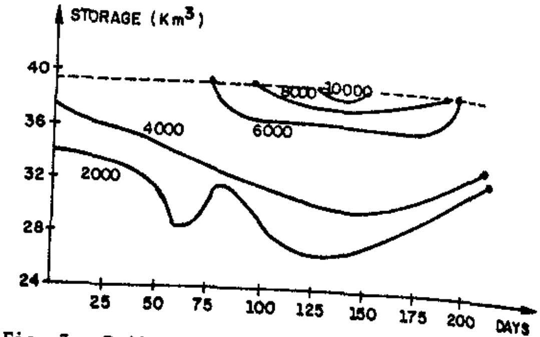

Fig. 3. Daily Operating Policy for Sobradinho Additive Flooding Costs, Case 4

CONCLUSIONS

An alternative approach for the traditional Stochastic Dynamic Programming method is presented, that may be applied when the inflow process considered is exceedingly complex, as is the case with daily flows. This approach is called here Sampling Dynamic Programming, and consists of keeping track of each sequence of the record individually, in order to estimate mean costs of operation. It is applied to the daily operation of a reservoir with flood control and energy-generation purposes, and the impact ever-greater flooding costs on the operating policy is verified by a sensitive analysis. Coupling of daily operation with monthly operation is obtained by the boundary condition for the costs of operation at the end of the flood season and by the cost of energy-deficits, both of with are supplied by the long range planning developed on a monthly basis.

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